# Race to the Top in Traffic Calming

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*Abstract*
We study the competition of two suburbs that are facing transit traffic flows. We show that in the absence of toll measures, the Nash equilibrium leads to a race to the top in traffic calming, except for the measures that do not affect the generalized cost of traffic. The Nash equilibrium is compared to two types of centralized decisions: the symmetric solution and the asymmetric solution. It is shown how the asymmetric solution that concentrates all transit traffic in one suburb is better but can only be realized if the authority over the local roads is transferred to the central authority.

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**Keywords:**

Transport, Externalities, Traffic calming; Multi-level government, Regulation

**JEL-codes:**

R48; R41;Q58;H23;H77;

# 1   Introduction

Many local communities struggle with the nuisance of transit traffic. Trucks and cars that have nothing to do with the local community cause noise, air pollution and accident damage to the inhabitants. In some cases they also delay local traffic. These are clearly external costs and in theory one could implement tolling systems that charge for the negative externalities, people could relocate away from the externalities etc. In practice we see a much more down to earth approach, including measures like speed bumps, traffic lights, pedestrian overpasses and noise barriers but also by shifting responsibilities where local roads that suffer from intensive transit traffic are transferred into the national road network.

The optimal design and the effects of these traffic measures have been studied intensively by engineers but the economics and the role of political institutions have been somewhat neglected. In this paper we study the problem of two suburbs that can reduce the negative externalities of transit traffic by taking different types of mitigating measures. As transit can choose its route, the two suburbs compete in discouraging transit traffic and end up in a race to the top where an excessive number of mitigating measures (such as speed bumps) are used. This race to the top only exists for some types of measures but not for all. Measures like noise barriers and pedestrian overpasses or tunnels are installed in the right intensity by the local authorities but this is no guarantee for an efficient solution. For the abatement of some negative externalities, there are increasing returns to scale and this means it can be interesting to concentrate transit traffic into one road or one suburb. But this is only possible if a higher government level steps in and takes full control on transit traffic flows. This is precisely what traffic engineers do when they propose a hierarchy of roads, with some roads designed for concentrating transit traffic.

Section 2 reviews the literature, section 3 sets out the elements of the model, section 4 analyses the behavior of local and national governments for three different measures. Section 5 contains a numerical illustration of the model and section 6 concludes.

# 2   Literature Review

De Borger and Proost (2013a) considered traffic calming measures where either the local government or the federal government can take the initiative. They make a typology of measures and show how measures that increase the generalized cost of transit traffic are used excessively by local governments. Compared with our paper, De Borger and Proost (2013a) look into the behavior of one local government while we focus on the horizontal competition between two local governments.

The horizontal tax competition between two governments for transit traffic is studied in De Borger, Proost and Van Dender (2005) and surveyed in De Borger and Proost (2012). They consider two localities that each offer a route for transit traffic. Traffic generates congestion but can also be taxed and this leads to too high road charges. When the two transit route alternatives are good substitutes, the price competition between the two alternatives limits the road charges and this type of competition is welfare improving. Our paper focuses on non-pricing measures and this type of competition will often prove to be detrimental to welfare.

Optimal pricing in transit countries is studied in van der Loo and Proost (2013) and De Borger and Proost (2013b), Westin, Franklin, Grahn-Voorneveld and Proost (2012) and Besley and Coate (2003) all use a political economy approach to study differences between centralized and decentralized policies. Compared to these papers we focus on non-pricing measures.

Horizontal competition between local governments that want to avoid the location of a negative externality is an important issue in environmental economics. There is a long standing debate on a possible race to the bottom where jurisdictions use taxes on mobile capital as well as regulations to maximize local welfare. Oates and Schwab (1988) show what outcome to expect in a world with many jurisdictions competing for capital that brings higher wages but also local pollution. They show that a tax on capital, combined with a standard on pollution can generate a first best solution if jurisdictions are homogeneous. Levinson (2002) shows that constraints on policy instruments often make this ideal impossible and this can give rise to a race to the bottom in standards or taxes and even a race to the top and Levinson (1999) shows that the inefficiency depends on the tax elasticity of the polluters' responses. Compared to this literature, transit traffic is like mobile capital bringing negative externalities that cannot be taxed, so our race to the top is in line with this literature.

All traffic calming or externality reducing measures like speed bumps, roundabouts, speed tables, roadway striping that narrow traffic lanes and encourage lower traffic speeds, new traffic lights, etc. have been intensively studied in the engineering and urban planning literature (see, e.g., Harvey (1990), Elvik (2001), Ewing (1999, 2001), Ewing and Brown (2009), Holzinger, Knill and Sommerer (2008) and Kahn and Goedecke (2011))[2]. The available studies suggest that such investments can be quite effective at reducing speeds: 15% to 40% and they can also reduce accident risks (12 to 45%) and local pollution. The engineering and accident analysis studies focus mostly on the costs and the effect of the traffic calming measures on noise, speed, accidents etc. for a given traffic volume. The

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[2] The use of traffic calming measures is widespread in European cities but also in several US cities (including well known examples such as Seattle, San Francisco and Portland). For detailed accounts of the effects of some of these measures on speeds, traffic volumes, crashes, etc. see the website of the Portland Bureau of Transportation (www.portlandoregon.gov/transportation).

relocation of traffic as a result of these measures is often not considered. This relocation and horizontal effect is also not studied in the scarce economic assessments of traffic calming measures. Garrod, Scarpa and Willis (2002) studied the WTP for traffic calming in British towns but these studies focus on a given policy implementation and not on what policies result from the policy process.

# 3   The analytical model

Our model is a combination of De Borger, Proost and Van Dender ("DBPVD",2005) and De Borger and Proost ("DBP", 2013a). DBPVD (2005) analyze the tolling of a parallel network that is used by local traffic and by transit traffic. We use the same type of parallel network but replace the tolls as policy instruments by different types of traffic calming measures. For the characterization of the traffic calming measures, we use the typology of DBP (2013a).

## 3.1   The network

We consider two suburbs, labeled A and B. A road passes through each suburb, the road is used by both local traffic and transit traffic. Transit traffic can choose between both roads while local traffic cannot change road. To simplify the model we assume that each suburb has a homogenous population where all citizens are drivers. The demand for travel by local users is $Y_A$ and $Y_B$ and the demand for through traffic is $X_A$ and $X_B$. We assume that the demand for local travel $Y_A$ and $Y_B$ are fixed and that the total demand for transit traffic is fixed but its allocation over the network is not, i.e. $X_A + X_B = 2X$. To simplify the analysis we only consider a symmetric situation where $Y_A = Y_B = Y$. We discuss the generalization to traffic volumes that are price dependent in our concluding section. An illustration of the transport network is shown in Figure 1.
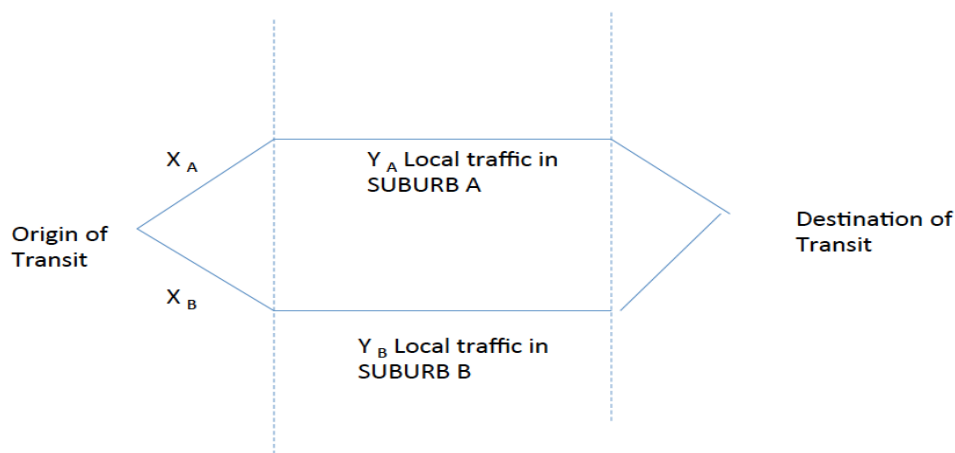


*Figure 1: Transport network with two parallel roads and two suburbs.*

## 3.2   External costs and other costs

All traffic in a suburb or city gives rise to a local external cost. We use a simplified version of the set up of De Borger and Proost (2013a). The total external cost $E_i$ (from emission of noise, pollution, accident risks, etc.) in area $i \in \{A, B\}$ is a linear function of the total amount of traffic on road $i$, i.e.

$$E_i = e(1 - az_i)(X_i + Y) \tag{1}$$

where $e$ is the emission level, $z_i$ is the level of investment in externality-reduction in suburb $i$ and $a$ captures the efficiency of the investment. The investment is normalized such that an investment $z$ reduces the externality from $e$ to $e(1 - az)$. We assume that the emission abatement cost has constant marginal cost curves in $z_i$, i.e. $\frac{dE_i}{dz_i} = e(1 - az_i)\frac{dX_i}{dz_i} - ea(X_i + Y)$. We only consider externality measures such that $E_i \geq 0$, i.e. $0 \leq z_i \leq \frac{1}{a}$.

We classify externality reduction measures of a given efficiency $a$ by two types of costs: a cost to the local government and a cost to the drivers that use the network. The cost, for the local government budget, of an investment in externality reduction $z_i$ in suburb $i$ is:

$$C_i(z_i) = \frac{b}{2}z_i{}^2 \tag{2}$$

This cost consists of investment and maintenance costs but can also consist of enforcement costs for speed restrictions etc. The second type of cost associated to an externality reducing measure is an increase in the generalized cost for local and transit travelers. Hence, the generalized cost for the local and transit travelers on road $i$ is a function of the investment in traffic calming $z_i$:

$$G_i(z_i) = c + \frac{d}{2}z_i^2 \tag{3}$$

where $c$ is the generalized (money plus time) user cost in the absence of externality-reducing investment and $i \in \{A, B\}$. We could also include congestion but as we will show later this would hardly affect our main results.

## 3.3   The allocation of transit traffic over the network

To allocate transit traffic over the network we analyze two different network allocation assumptions. First we analyze a situation where the distribution of transit traffic follows the Wardrop principle: all transit travelers use the route with the lowest generalized cost. To simplify the analysis we assume that the transit traffic is not fully homogeneous (transit drivers have different access distance at origin or destination) so that we get a smooth distribution of transit traffic over the two routes. We assume that the distribution of transit traffic is a linear function of the difference in generalized cost between the two routes, i.e. $G_i(z_i) - G_j(z_j) = d(z_i - z_j)$, with a slope parameter $\Delta$:

$$X_i = \begin{cases} 0 & \text{if} & d(z_i - z_j) \geq \Delta \\ X\left(1 - \frac{d(z_i - z_j)}{\Delta}\right) & \text{if} & -\Delta < d(z_i - z_j) < \Delta \\ 2X & \text{if} & d(z_i - z_j) \leq \Delta \end{cases}$$
$$X_j = 2X - X_i$$

(4)

If $\Delta \to 0$ we have homogeneous transit traffic. Assuming an interior solution, the derivatives of $X_i$ with respect to a change in $z_i$ and $z_j$ are $\frac{dX_i}{dz_i} = -X\frac{d}{\Delta}$ and $\frac{dX_i}{dz_j} = X\frac{d}{\Delta}$ respectively.

Next to the Wardrop principle, we also analyze a network allocation where the federal government can direct transit traffic using a traffic distribution parameter $\beta$. This parameter can correspond to traffic signs, road blocks or other measures that affect the allocation of transit traffic without altering the generalized cost on the local roads. When the federal government directs transit traffic directly using the second principle, the allocation of transit traffic is:

$$X_i = 2X\beta$$
$$X_j = 2X(1 - \beta)$$

(5)

where $0 \leq \beta \leq 1$ and $\frac{dX_i}{dz_i} = \frac{dX_i}{dz_j} = 0$.

## 3.4   The behavior of governments

There are two types of governments: one federal government that represents the interest of transit traffic, local traffic and residents in both suburbs and two local governments who represent local traffic and residents in their own suburb only. Inhabitants are not mobile. The local governments can only invest in their own suburb while the federal government can control investments in both suburbs. The federal government also has the option to direct transit traffic directly using a traffic distribution parameter $\beta$. We assume that the local governments want to maximize welfare in their local suburb while the federal government wants to maximize total welfare. These assumptions make sense when each of the two suburbs are homogeneous and when the federal government decides on the basis of a bargaining process. Welfare for suburb $i \in \{A, B\}$ is then the sum of total transport costs of the local users minus the costs of traffic calming investments by the local governments minus the local externality cost, that is:

$$W_i = -\left(c + \frac{d}{2}z_i^2\right)Y - \frac{b}{2}z_i^2 - e(1 - az_i)(X_i + Y)$$

(6)

We assume that each local government takes the behavior of the other government as given so that we can look for a Nash equilibrium. In the general setting, the behavior of the local government can influence the amount of transit traffic by increasing the generalized cost of using the road, i.e.

$\frac{dX_i}{dz_i} \leq 0$ is the change in transit traffic $X_i$ from an increase in $z_i$. The first-order-condition for the local government in suburb $i$ is:

$$\frac{dW_i}{dz_i} = -Y dz_i - b z_i + ea(X_i + Y) - e(1 - az_i)\frac{dX_i}{dz_i} = 0 \tag{7}$$

The welfare function for the federal government also includes the transport costs of transit traffic:

$$\begin{aligned} W_F = &- \left(c + \frac{d}{2}z_i^2\right)(X_i + Y) - \left(c + \frac{d}{2}z_j^2\right)(X_j + Y) - \frac{b}{2}z_i^2 - \frac{b}{2}z_j^2 \\ &- e(1 - az_i)(X_i + Y) - e\left(1 - az_j\right)(X_j + Y) \end{aligned} \tag{8}$$

The first-order-conditions for the federal government are:

$$\begin{aligned} \frac{dW_F}{dz_i} = &-(X_i + Y)dz_i - \left(c + \frac{d}{2}z_i^2\right)\frac{dX_i}{dz_i} - \left(c + \frac{d}{2}z_j^2\right)\frac{dX_j}{dz_i} - bz_i \\ &+ ea(X_i + Y) - e(1 - az_i)\frac{dX_i}{dz_i} - e\left(1 - az_j\right)\frac{dX_j}{dz_i} = 0 \\ \frac{dW_F}{dz_j} = &-(X_j + Y)dz_j - \left(c + \frac{d}{2}z_j^2\right)\frac{dX_j}{dz_j} - \left(c + \frac{d}{2}z_i^2\right)\frac{dX_i}{dz_j} - bz_j \\ &+ ea(X_j + Y) - e\left(1 - az_j\right)\frac{dX_j}{dz_j} - e(1 - az_i)\frac{dX_i}{dz_j} = 0 \end{aligned} \tag{9}$$

where $\frac{dX_i}{dz_i} \leq 0$, $\frac{dX_j}{dz_j} \leq 0$, $\frac{dX_i}{dz_j} \geq 0$ and $\frac{dX_j}{dz_i} \geq 0$ depend on which network allocation assumption is being used for distributing transit traffic between the two suburbs.

Our formulation of traffic calming measures allows for many types of measures. De Borger and Proost (2013a) showed the important difference between traffic calming measures that do not affect the generalized costs of transit traffic and those that do affect the transport user costs. For this reason we discuss both measures separately. We start with the measures that do not affect the generalized costs of transit traffic: noise barriers, pedestrian overpasses etc. The outcome for this type of measure is then compared to the outcome where measures are used that do affect the transport costs on the local roads: speed bumps, speed restrictions, traffic lights etc.

The measures are characterized by three key parameters:

- Parameter $a$ captures the effectiveness of externality reduction measures $z$
- Parameter $b$ captures investment cost for public budget associated with $z$
- Parameter $d$ captures how the investment $z$ affects the generalized cost of a trip

For example, quiet asphalt, noise barriers and pedestrian overpasses correspond to $b > 0$ and $d = 0$; speed bumps to $b > 0$ and $d > 0$; and speed restrictions to $b = 0$ and $d > 0$. We also assume that $a > 0, b \geq 0, c \geq 0, d \geq 0, e \geq 0, X_i \geq 0, Y \geq 0$ and that $0 \leq z_i \leq \frac{1}{a}$ for $i \in \{A, B\}$.

# 4   Analysis

We start with measures like noise walls and pedestrian overpasses. We then continue by analyzing speed bumps and speed restrictions. For each of these measures we study whether the local government and federal government make the same choices. Next we examine whether the federal government can do better by taking full control of the network and directing all transit traffic to one of the suburbs and analyze the effect on local welfare from an asymmetric solution. This section is concluded by analyzing the transfers that are needed to make the asymmetric solutions acceptable.

## 4.1   Noise walls and pedestrian overpasses

We start to analyze noise walls and pedestrian overpasses by comparing the optimal level of externality reduction under centralized and decentralized decision making. Noise walls and pedestrian overpasses reduce the external cost without affecting the generalized cost of the travelers, hence $d = 0$ and $\frac{dX_i}{dz_i} = 0$.

**Proposition 1:** *Given the volume of transit traffic, the local government will invest in the optimal level of externality reduction measures when these measures do not affect the user cost of transport.*

Consider first the welfare for the local government in suburb $i \in \{A, B\}$:

$$W_i = -Y_i c - \frac{b}{2}z_i^2 - e(1 - az_i)(X_i + Y) \tag{10}$$

Calculating the first-order-condition for the local government in suburb $i$ and solving for the optimal level of externality reduction we get:

$$z_i^{local} = \frac{ea(X_i + Y)}{b} \tag{11}$$

When using noise walls, the optimal level of externality reduction $z_i$ chosen by the local government in suburb $i$ does not depend on the behavior of the local government in suburb $j$. With $d = 0$, the welfare function for the federal government becomes:

$$W_F = -(X_i + Y)c - (X_j + Y)c - \frac{b}{2}z_i^2 - \frac{b}{2}z_j^2$$
$$-e(1 - az_i)(X_i + Y) - e(1 - az_j)(X_j + Y) \tag{12}$$

The first-order-condition for the federal government for $i \in \{A, B\}$ is:

$$\frac{W_F}{dz_i} = -bz_i + eaz_i(X_i + Y) = 0 \tag{13}$$

Solving for the federal optimum we get:

$$z_i^{federal} = \frac{ea(X_i+Y)}{b} \tag{14}$$

The optimal level of noise reduction is hence identical to the solution chosen by the local governments, given that the government has no measure to direct traffic between the suburbs. Observe that (11) and (14) only hold if $z_i \leq \frac{1}{a}$, i.e. $X_i + Y \leq \frac{b}{ea^2}$. If the externality damage is high and the cost of reducing the externality very low, the optimal noise wall will completely eliminate the externality damage. This result is due to De Borger and Proost (2013a).

## 4.2    Speed bumps

Next we consider speed bumps. This implies that $d > 0$ and $b > 0$. We first calculate the federal optimum and then compare it to the solution preferred by the local governments. The federal government wants to maximize total welfare $W_F$ in equation (8).

Assume first that the government cannot control transit traffic directly using traffic signs and that we have a symmetric solution where $z_i = z_j$, $X_i = X_j = X$ and $\frac{dX_i}{dz_i} = \frac{dX_i}{dz_j} = 0$. Solving the first-order-condition in (9) for the optimal investment level we get:

$$z_i^{federal} = \frac{ea(X_i+Y)}{(X_i+Y)d+b} \tag{15}$$

A question is now whether the federal government can increase welfare by shifting transit traffic to one of the suburbs, i.e. by choosing $z_i \neq z_j$. Assume that $z_j > z_i$ such that transit traffic is diverted from suburb $j$ to suburb $i$, i.e. $X_i > X_j$. Since

$$\frac{ea(X_i + Y)}{(X_i + Y)d + b} > \frac{ea(X_j + Y)}{(X_j + Y)d + b}$$

the federal optimum would be to set $z_j < z_i$. Since it is optimal for the federal government to invest more in the suburb with the highest share of traffic, while at the same time speed bumps direct traffic towards the suburb with the least investments, a corner solution is not feasible. With no direct control of the distribution of transit traffic, it is thus optimal for the federal government to choose a symmetric solution where $X_i = X_j = X$ by having the same number of speed bumps in both suburbs, i.e.

$$z_i^{federal} = z_j^{federal} = \frac{ea(X+Y)}{(X+Y)d+b} \tag{16}$$

The symmetric federal optimum can be compared to the investment level preferred by local decision makers in each suburb. We assume that the local government in suburb $i$ takes the investment in

suburb $j$ as given when choosing its traffic calming level $z_i$. We calculate the response functions for the local governments and see if the local governments prefer to divert from the federal solution.

***Proposition 2****: The local governments install more speed bumps than the federal government.*

Assuming an interior solution, we can solve the first-order-condition for the local government in equation (7) for the optimal investment level to get:

$$z_i^{local} = \frac{ea(X_i+Y)-e\frac{dX_i}{dz_i}}{Yd+b-ea\frac{dX_i}{dz_i}} \tag{17}$$

Since $\frac{dX_i}{dz_i} \leq 0$ the local government have an incentive to invest more in speed bumps than the federal optimum in order to try to divert transit traffic to the other suburb. To prove this we assume that we have a symmetrical situation where both local governments invest the same amount in speed bumps and assume that distribution of transit traffic is given by (4). Hence $z_i = z_j = z$ and:

$$z_i^{local} = \frac{ae(X_i+Y)+eX_i\frac{d}{\Delta}}{Yd+b+eaX_i\frac{d}{\Delta}} > \frac{ea(X_i+Y)}{(X_i+Y)d+b} = z_i^{federal} \tag{18}$$

Note that if $d = 0$ the two expressions becomes identical as shown in the example with noise walls. This result is in line with findings of DBP, 2013a who considered a problem with only one local government.

***Proposition 3****: If the transit traffic is very elastic, the local government has an incentive to invest in traffic calming measures to remove the negative externality completely.*

The difference between $z_i^{local}$ and $z_i^{federal}$ depends both on the derivative $\frac{dX_i}{dz_i}$ and the absence of consideration for the welfare of the transit travelers. Assume that the distribution of transit traffic follows the Wardrop principle as in equation (4). When the slope parameter $\Delta$ is very large (corresponding to a situation where transit traffic is very inelastic)

$$\lim_{\Delta \to \infty} z_i^{local} = \frac{ea(X_i+Y)}{Yd+b} \tag{19}$$

When transit traffic is very elastic ($\Delta \to 0$) the preferred investment by the local government becomes:

$$\lim_{\Delta \to 0} z_i^{local} = \frac{1}{a} \tag{20}$$

Hence, when the slope parameter $\Delta$ goes to zero, the local governments prefer to invest in traffic calming until the negative externality of traffic is completely eliminated.

## 4.3   Speed restrictions

Speed restrictions can be seen as a special case of speed bumps where the government can reduce the negative externality by lowering the speed. Assuming that the measure does not require increased enforcement, the additional cost for the government of changing the speed is almost zero, i.e. $d > 0$ and $b = 0$. Assuming a symmetric solution and solving the first-order-condition for the federal government's welfare maximization problem (9) we see that the federal government would set the traffic calming level $z_i$ to:

$$z_i^{federal} = \frac{ea}{d} \tag{21}$$

From equation (17) we see in a similar way that the local government in suburb $i$ would set:

$$z_i^{local} = \frac{ea(X_i+Y)-e\frac{dX_i}{dz_i}}{Yd-ea\frac{dX_i}{dz_i}} \tag{22}$$

where $\frac{dX_i}{dz_i} \leq 0$. Since $z_i^{federal} < z_i^{local}$ the local government has an incentive to reduce speed more than the federal government.

## 4.4   Effects on federal welfare of an asymmetric solution

Can the federal government do better by deviating from the symmetrical solutions shown above? In the calculations above we have assumed that the federal government can only direct traffic by differentiating the generalized cost of travel on the two routes, i.e. by setting $d(z_i - z_j) \neq 0$. As long as transit traffic can choose its own route, the federal government is forced to choose a symmetric solution with equal investments in both suburbs.

We now assume that the federal government can instead control directly the allocation of transit traffic between the two suburbs using a parameter $\beta$.

***Proposition 4**: The federal government prefers to direct all transit traffic to the suburb with most local traffic as long as there is an investment cost for the government.*

Assume that the federal government controls the distribution of transit traffic directly using a network allocation parameter $\beta$. By inserting the expressions for transit traffic in equation (5) into the federal welfare function (8) we get:

$$W_F^{\beta} = -\left(c + \frac{d}{2}z_i^2\right)(2X\beta + Y) - \left(c + \frac{d}{2}z_j^2\right)(2X(1-\beta) + Y) - \frac{b}{2}z_i^2 - \frac{b}{2}z_j^2$$
$$-e(1-az_i)(2X\beta + Y) - e(1-az_j)(2X(1-\beta) + Y) \tag{23}$$

Since transit traffic is directly controlled by the federal government, $\frac{dX_i}{dz_i} = 0$ and the first-order-conditions for the federal government become:

$$\frac{dW_F^\beta}{dz_i} = -(2X\beta + Y)dz_i - bz_i + ea(2X\beta + Y) = 0$$

$$\frac{dW_F^\beta}{dz_j} = -(2X(1-\beta) + Y)dz_j - bz_j + ea(2X(1-\beta) + Y) = 0$$

(24)

Solving for the optimal investment level as function of $\beta$ we get:

$$z_i^\beta = \frac{ea(2\beta X + Y)}{(2X\beta + Y)d + b}$$

$$z_j^\beta = \frac{ea(2(1-\beta)X + Y)}{(2X(1-\beta) + Y)d + b}$$

(25)

We prove in the Appendix that $W_F^\beta$ is strictly convex in β so the federal welfare maximum is always a corner solution where all transit traffic is directed to one of the suburbs. Because the suburbs are identical, the federal government is indifferent to which suburb transit traffic is directed to.

The net welfare gain for the federal government from directing all transit traffic to one suburb is:

$$\Delta W_F^{\beta=1} = \frac{e^2 a^2 b^2 X^2}{(b + Yd)(2X^2 d^2 + 3XY d^2 + 3Xbd + Y^2 d^2 + 2Ybd + b^2)} \geq 0$$

(26)

where $\Delta W_F^{\beta=1}$ is the difference between federal welfare when all transit traffic is directed to a single suburb ($\beta = 1$ or $\beta = 0$) and federal welfare when transit traffic is equally distributed between the suburbs ($\beta = \frac{1}{2}$).

For noise wall type of measures (with $d = 0$), the net welfare gain from an asymmetric solution compared to a symmetric solution can be shown graphically. If all transit traffic is directed to suburb $i$ (and $d = 0$), the welfare maximizing levels of traffic calming are equal to $z_i = \frac{ea(2X+Y)}{b}$ and $z_j = \frac{eaY}{b}$. When transit traffic is symmetrically distributed, the optimal investment levels are $z_i = z_j = \frac{ea(Y+X)}{b}$. The welfare gains of the two situations are illustrated in Figure 2 below:
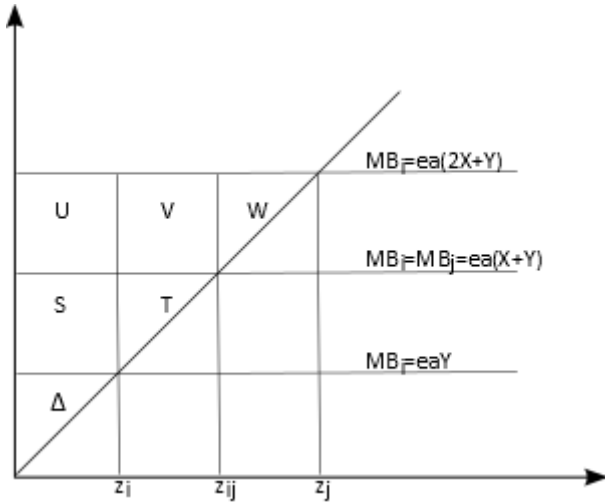
*Figure 2: Graphical illustration of the welfare gain of an asymmetric solution.*

The welfare gain is equal to the areas $U + V + W + S + T + \Delta$ (total of saved external costs − total cost in suburb B) $+\Delta$ (net benefit in suburb A) $-2(S + T + \Delta)$ (net benefit in case transit traffic is spread). As by construction, $U = S$ and $T = W$, the net advantage of the asymmetric solution is equal to area:

$$V = eaX \cdot \frac{eaX}{b} = \frac{e^2 a^2 X^2}{b} \tag{27}$$

Equation (27) can also be obtained by setting $d = 0$ in equation (26).

This result is due to the economies of scale built into almost all externality reduction investments. Noise walls are an obvious case. In our formulation, one noise wall will take away a given proportion $az$ of all the noise generated by each car passing the noise wall. So it is more efficient to use each noise wall for a maximum number of cars and this means sending all transit traffic to only one suburb. In this suburb one invests more in noise walls than in the other suburb with only local traffic. For measures of the speed bump type, this still holds as there is still a government investment needed. It is only when we consider speed restrictions for which there is, with our assumptions, no government investment needed ($b = 0$) that the concentration of transit traffic in one suburb does not generate a welfare gain.

## 4.5   Effects on local welfare from an asymmetric solution

We continue by analyzing the effects on the two suburbs from an asymmetric solution where all transit traffic is directed to a single suburb using a network allocation parameter $\beta$. We calculate the difference in local welfare in suburb $i$ and $j$ when the federal government directs all transit traffic to suburb $i$ and sets the traffic calming levels in the two suburbs to maximize total welfare.

Assume that the federal government directs all transit traffic to suburb $i$, i.e. $X_i = 2X$ and $X_j = 0$. The optimal level of traffic calming in the suburbs are hence:

$$z_i = \frac{ea(2X+Y)}{(2X+Y)d+b}$$
$$z_j = \frac{eaY}{Yd+b}$$

(28)

Inserting these expressions into the welfare functions for the local decision makers we get:

$$W_i^{\beta=1} = -\left(c + \frac{d}{2}\left(\frac{ea(2X+Y)}{(2X+Y)d+b}\right)^2\right)Y - \frac{b}{2}\left(\frac{ea(2X+Y)}{(2X+Y)d+b}\right)^2$$
$$-e\left(1 - a\frac{ea(2X+Y)}{(2X+Y)d+b}\right)(2X+Y)$$
$$W_j^{\beta=1} = -\left(c + \frac{d}{2}\left(\frac{eaY}{Yd+b}\right)^2\right)Y - \frac{b}{2}\left(\frac{eaY}{Yd+b}\right)^2$$
$$-e\left(1 - a\frac{eaY}{Yd+b}\right)Y$$

(29)

The local welfare for the asymmetric situation can be compared to the symmetric situation when transit traffic is equally distributed between the two suburbs, i.e. $X_i = X_j = X$ and $z_i = z_j = \frac{ea(X+Y)}{(X+Y)d+b}$:

$$W_i^{\beta=\frac{1}{2}} = W_j^{\beta=\frac{1}{2}} = -\left(c + \frac{d}{2}\left(\frac{ea(X+Y)}{(X+Y)d+b}\right)^2\right)Y$$
$$-\frac{b}{2}\left(\frac{ea(X+Y)}{(X+Y)d+b}\right)^2 - e\left(1 - a\frac{ea(X+Y)}{(X+Y)d+b}\right)(X+Y)$$

(30)

To simplify the calculations we only consider the special case when $d = 0$ and calculate the net welfare effect on the local government in suburb $i$ and $j$ from an asymmetric solution:

$$\Delta W_i^{\beta=1} = W_i^{\beta=1} - W_i^{\beta=\frac{1}{2}} = \frac{e^2 a^2}{2b}X(3X+2Y) - eX$$
$$\Delta W_j^{\beta=1} = W_j^{\beta=1} - W_j^{\beta=\frac{1}{2}} = -\frac{e^2 a^2}{2b}X(X+2Y) + eX$$

(31)

Equation (31) gives the transfers needed to mace the asymmetric solution acceptable for a suburb when $d = 0$. From the equations we see that the sign of the net welfare effect from receiving transit traffic is ambiguous. The change in local welfare for a suburb from receiving transit traffic can be divided into two effects. The first effect is the direct effect on local welfare from receiving more transit traffic, this effect is always negative. The second effect is the change in local welfare from allowing the federal government to choose the optimal level of traffic calming. From equation (18) we see that the local government prefers more traffic calming than the federal government. In some situations, this means that even though the local suburb loses on receiving transit traffic, the negative effect is outweigh by the fact that the federal government chooses a better level of traffic calming in the asymmetric situation than in the symmetric situation. However, if the local

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governments choose traffic calming level, the suburb that receives all the transit traffic is always worse off.

**Proposition 5**: *For a given level of traffic calming, a suburb receiving the transit traffic is always worse off.*

To show this, we consider the net welfare effect on the local government in $i$ from an asymmetric solution as a function of the traffic calming level $z_i$:

$$\Delta W_i^{\beta=1}(z_i) = W_i^{\beta=1}(z_i) - W_i^{\beta=\frac{1}{2}}(z_i) = -e(1 - az_i)X \leq 0 \tag{32}$$

Since $W_i^{\beta=1}(z_i) \leq W_i^{\beta=\frac{1}{2}}(z_i)$ for $0 \leq z_i \leq \frac{1}{a}$ it follows that $\max_{z_i} W_i^{\beta=1}(z_i) \leq \max_{z_i} W_i^{\beta=\frac{1}{2}}(z_i)$.

As the federal welfare is, in our model the sum of the welfares of the citizens of the two suburbs and transit traffic, there is a bargaining solution where the two suburbs agree to concentrate transit traffic in one suburb and where one suburb finances part of the measures in the other suburb. When the federal government takes control of the transit traffic by manipulating β we end up in a system of hierarchical roads where the local government in the transit suburb gives up its control on the externality abatement measures. In that system it is obviously crucial that the local government is no longer allowed to invest in any measure of the speed bump type that could send transit traffic again to the other suburb.

## 4.6   Noise walls versus speed bumps

In the final section in the analysis we study how the preferences between different types of measures differs between local and federal decision makers. To do this, we study a situation where the federal government is indifferent between using noise walls type measures (say pedestrian overpass) and speed bumps. What do local decision makers prefer if federal government is indifferent? To simplify the calculations we assume symmetry and only consider the effect in one suburb.

Assume that the federal government chooses between two different measures, speed bumps (d) and noise walls (0). The speed bumps alternative has the parameters $(a, b, d)$ and the alternative with noise walls has the parameters $(a', b', 0)$. From equation (16) we get the optimal levels of traffic calming $z_d$ and $z_0$:

$$\begin{aligned} z_d &= \frac{ea(X+Y)}{(X+Y)d+b} \\ z_0 &= \frac{ea'(X+Y)}{b'} \end{aligned} \tag{33}$$

Using these the optimal levels of traffic calming, federal welfare becomes:

$$W_d^{federal} = -\left(c + \frac{d}{2}z_d{}^2\right)(X+Y) - \frac{b}{2}z_d{}^2 - e(1-az_d)(X+Y)$$
$$W_0^{federal} = -c(X+Y) - \frac{b'}{2}z_d{}^2 - e(1-a'z_0)(X+Y)$$

(34)

Assuming that the parameters $a$, $b$, $d$, $a'$, $b'$ are chosen such that $W_0^{federal} = W_d^{federal}$, we can calculate the difference in local welfare between the two alternative measures. Comparing equation (6) and (8) we get that:

$$W^{local} = W^{federal} + \left(c + \frac{d}{2}z^2\right)X$$

(35)

This implies that local welfare is higher than federal welfare for a given situation. Since we only measure costs, both local and federal welfare are negative. For any given level of $z$, the local government only cares about the cost of local traffic while the federal government also considers the cost for transit traffic. Hence the welfare for the local government is less negative than the welfare for the federal government.

Using the difference above we find that:

$$\Delta W^{local} = W_0^{local} - W_d^{local} = -\frac{d}{2}\left(\frac{ea(Y+X)}{(Y+X)d+b}\right)^2 X < 0$$

(36)

Since $W_0^{local} < W_d^{local}$, the local governments prefer speed bump type of measures to noise walls when the federal government is indifferent. The reason is that, everything else equal, the cost of the speed bump solution is partly borne by the transit traffic and this is not included in the local governments' objective function.

There are two types of solutions to this. First one can give the control on particular roads with a lot of traffic to a higher government level and this government level will favor a solution where one concentrates the transit traffic flows in one suburb. The second solution is to leave the control of the roads to the suburbs but add regulations and taxes or subsidies.

Setting up the right combination of federal controls is not easy. The easier type of regulation is a maximum number of speed bump type of measures (empirical evidence?) that a local government can install. This regulation can bring us closer to the symmetric federal optimum but not to the federal optimum where transit traffic is concentrated. Alternatively, one can tax speed bump measures or subsidize noise walls but the latter measure will give rise to an excessive number of noise walls. It is the non-symmetric optimum solution that is difficult to decentralize.

# 5   Numerical illustrations

To illustrate how the local and federal optimum depend on the type of traffic calming, we study a small numerical illustration. Numerical values for the model parameters are given in Table 1.

*Table 1: Parameter values in the numerical illustration*

| Parameter | Value |
|:---:|:---:|
| $a$ | 0.5 |
| $b$ | 1 |
| $c$ | 1 |
| $d$ | [0, 0.01, 0.05, 0.2] |
| $e$ | 1 |
| $X$ | 1 |
| $Y$ | 1 |
| $\Delta$ | 0.01 |

Figure 3 shows the local and federal optimum for different types of traffic calming measures. The dotted and the solid lines show the reaction functions for the local governments in A and B. We compare the results for different values of $d$. This allows us to compare noise wall type of measures with speed bump type of measures ($d > 0$). For each value of $d$, we analyze the reaction functions, the Nash equilibrium as well as the federal symmetric optimum (shown by the square) and the federal asymmetric optimum (shown by the two circles). In table 2 we will discuss in more detail the asymmetric federal optimum.

For noise walls where $d = 0$, we see that the local optimum (given by the intersection of the local governments reaction functions) coincide with the symmetrical federal optimum. The reaction functions are horizontal as the optimal noise screen intensity does not depend on what the other suburb does because transit traffic does not react to noise screens. When the cost for the drivers increases, i.e. $d > 0$, the local optimum starts to deviate from the federal symmetrical optimum in a race to the top. Whenever $d > 0$, the Nash equilibrium has always more speed bumps than the federal optimum.
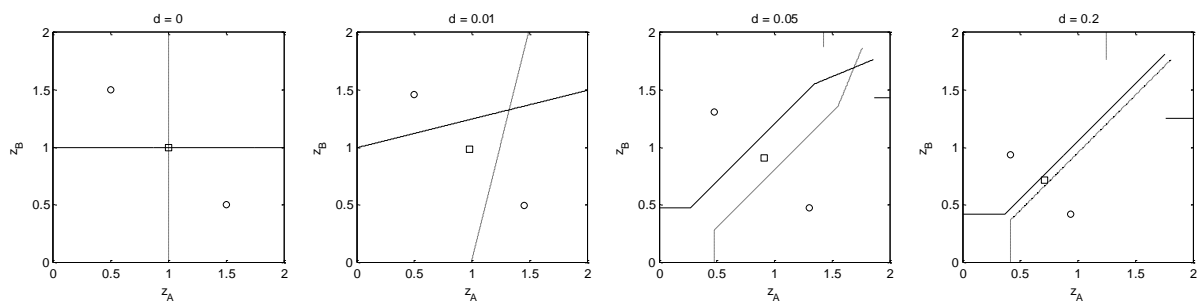


*Figure 3: Numerical illustration of response functions and federal optimum for different types of traffic calming measures*

However, if $d$ is very large there is no longer a unique solution since the reaction functions for the local governments do not intersect. To analyze this situation we analyze a numerical model where two players A and B take turn and adapt their investment level $z_i$ based on the investment level of the other player $z_j$. The result is shown in Figure 4.
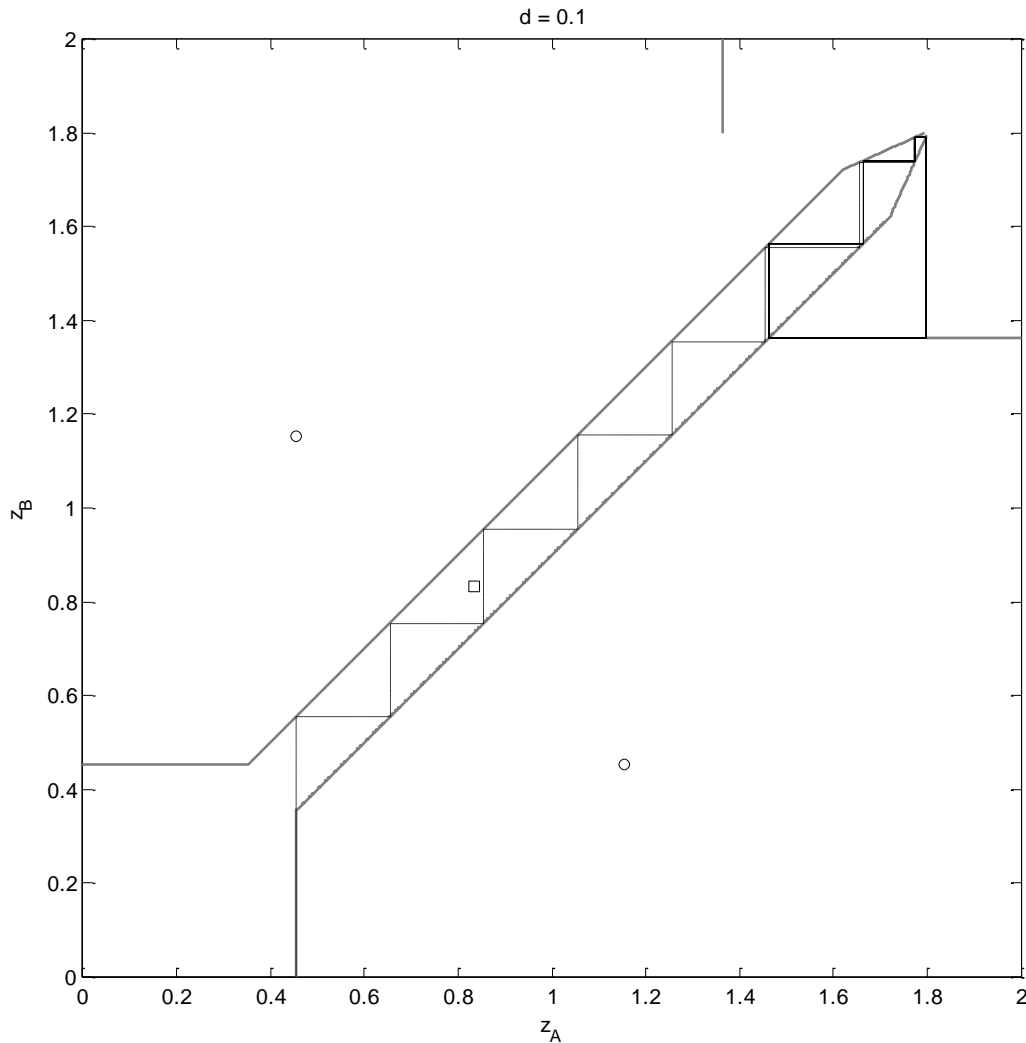


*Figure 4: Numerical simulation of race to the top with no stable solution*

In the figure, the grey lines show the response functions for player A and player B respectively. The dashed black line correspond to the optimal moves when the players take turn to adapt their investment level $z_i$ based on the other players previous investment level $z_j$. In the game, both players start with no traffic calming measures, i.e. $z_A = z_B = 0$. Player A begins, since player B has no traffic calming, it is optimal for player A to choose $z_A = 0.45$. In response to this, player B chooses $z_B = 0.55$. Player A responds by increasing the investment in traffic calming to $z_A = 0.65$ and so on in a race to the top where each player try to outspend the other player in order to shift transit traffic to the other player's suburb. However, when one of the players sets the investment level above a certain threshold (in this example $z > 1.795$) it is no longer optimal for the other player to continue

the race to the top. The best the player can do is to accept the defeat and set the investment level at a lower level. This does however trigger the other player to also lower his or her investment level and the race continues. Consider now also the comparison with the non-symmetrical federal solutions in Table 2.

*Table 2: Comparing symmetrical and asymmetrical optimums*

|  | $d = 0$ | $d = 0.01$ | $d = 0.05$ | $d = 0.2$ |
|---|---|---|---|---|
| **Traffic calming level $z_i$** |  |  |  |  |
| **Nash equilibrium (NE)** | 1.00 | 1.32 | 1.69 | 1.88 |
| **Federal symmetric optimum (FS)** | 1.00 | 0.98 | 0.91 | 0.71 |
| **Federal asymmetric optimum (FA)** | 1.50 / 0.50 | 1.46 / 0.50 | 1.30 / 0.48 | 0.94 / 0.42 |
| **(transit / no-transit)** |  |  |  |  |
| **Local welfare** |  |  |  |  |
| **Nash equilibrium (NE)** | -2.50 | -2.56 | -2.81 | -3.23 |
| **Federal symmetric optimum (FS)** | -2.50 | -2.50 | -2.52 | -2.59 |
| **Federal asymmetric optimum (FA)** | -2.88 / -1.88 | -2.89 / -1.88 | -2.94 / -1.88 | -3.12 / -1.90 |
| **Difference FS - NE** | 0.00 | 0.06 | 0.28 | 0.64 |
| **Difference FA - NE** | -0.38 / 0.63 | -0.33 / 0.69 | -0.13 / 0.93 | 0.11 / 1.34 |
| **Difference FA – FS** | -0.38 / 0.63 | -0.38 / 0.63 | -0.41 / 0.64 | -0.53 / 0.70 |
| **Total difference FA - NE** | 0.25 | 0.36 | 0.80 | 1.45 |
| **Total difference FA - FS** | 0.25 | 0.25 | 0.23 | 0.17 |
| **Federal welfare** |  |  |  |  |
| **Nash equilibrium (NE)** | -7.00 | -7.14 | -7.76 | -9.17 |
| **Federal symmetric optimum (FS)** | -7.00 | -7.02 | -7.09 | -7.29 |
| **Federal asymmetric optimum (FA)** | -6.75 | -6.78 | -6.90 | -7.19 |
| **Difference FS - NE** | 0.00 | 0.12 | 0.67 | 1.89 |
| **Difference FA - NE** | 0.25 | 0.36 | 0.86 | 1.98 |
| **Difference FA - FS** | 0.25 | 0.24 | 0.19 | 0.09 |

Graphically, the asymmetric solutions are shown in Figures 3 and 4 by the two circles, there are two asymmetric optima for each case. The detailed results are reported in Table 2. Of particular interest is the difference in welfare between the suburb accepting all the transit traffic and the other suburb. The total welfare for the two suburbs is larger in the asymmetric solution but the welfare of the "transit" suburb is lower than in the non-cooperative equilibrium, unless $d$ is very high when there is

a strong race to the top. This loss of welfare can be compared with the additional cost of the measures, one sees the loss of welfare is larger because not all the externalities of transit traffic have been removed.

# 6   Conclusions and caveats

We developed a simple model with two suburbs to show that local governments tend to take more traffic calming measures of the type that increase the costs for transit traffic. This creates a race to the top where all local governments end up using an excessive number of speed bumps in the hope of sending transit traffic to the other suburbs. This suboptimal policy can be corrected in two ways. First a higher level of government can regulate, tax or subsidize particular traffic calming measures: it can tax or put a cap on the number of traffic calming measures that increase the costs of transit traffic. Second it can take full control of some roads and concentrate all transit traffic in one suburb. This is a more cost efficient solution than a symmetric solution but will require transfers between the suburbs to make the policy acceptable.

This is a first exploration. There are many caveats and directions that still need to be explored.

First, we considered identical suburbs with homogeneous populations. Suburbs tend to be specialized in the sense that they attract people of similar income level, travel needs or other preferences. But suburbs are not homogenous: there will be people living closer to the main transit roads, some people are more sensitive for externalities etc. These differences are difficult to study. What is sure is that the political decision mechanism within each suburb becomes more important as the citizens will have different preferences. This will require a systematic analysis of the political decision making by distinguishing two groups per suburb (as in De Borger and Proost, 2013a) and comparing how these groups decide on centralized and on local level.

Second we left out traffic congestion. Unpriced traffic congestion is an additional source of externalities that each suburb tries to keep out its suburb. In this sense, each of the suburbs will even have more reasons to discourage transit traffic. Traffic congestion has a second dimension: it affects itself the flow of traffic: traffic will be spread nicely over the two suburbs if they have the same road width. This second dimension makes it more difficult to influence the distribution of traffic over two suburbs. Overall our conclusions tend to hold also in this type of setting except for the asymmetric solution that is somewhat self-defeating as concentrating more traffic on one suburb will increase congestion if road capacity is not adapted.

Third, we assumed that the total volume of traffic was not price elastic. Again this assumption will not change our main results. Transit (and local) traffic volumes will decrease and this may be an additional source of welfare loss when a race to the top is present.

Fourth, we studied one measure at a time or compared individual measures. In practice one will find that combinations of measures (noise walls and speed bumps) are used. As long as these measures address one externality only, our results tend to hold. Consider noise walls that address noise externalities and speed bumps that address accident externalities. There will still be an excessive number of speed bumps and a right amount of noise walls.

Finally, our results are confirmed by casual evidence and by the division of authority over roads in some countries but a more systematic empirical verification may reveal still other features.

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## Appendix

In the Appendix we prove that $W_F^\beta$ is strictly convex in $\beta$ and prove that the federal welfare maximum is always a corner solution where all transit traffic is directed to the suburb with the most local traffic. We also provide a more formal proof of equation (36).

### Non-symmetric situation

Inserting the optimal level of traffic calming $z_i^\beta$ and $z_j^\beta$ into the welfare function in (8) gives us:

$$W_F^\beta = -\left(c + \frac{d}{2}\left(\frac{ea(2\beta X+Y)}{(2X\beta+Y)d+b}\right)^2\right)(2X\beta+Y)$$
$$-\left(c + \frac{d}{2}\left(\frac{ea(2(1-\beta)X+Y)}{(2X(1-\beta)+Y)d+b}\right)^2\right)(2X(1-\beta)+Y) - \frac{b}{2}\left(\frac{ea(2\beta X+Y)}{(2X\beta+Y)d+b}\right)^2$$
$$-\frac{b}{2}\left(\frac{ea(2(1-\beta)X+Y)}{(2X(1-\beta)+Y)d+b}\right)^2 - e\left(1 - a\frac{ea(2\beta X+Y)}{(2X\beta+Y)d+b}\right)(2X\beta+Y)$$
$$-e\left(1 - a\frac{ea(2(1-\beta)X+Y)}{(2X(1-\beta)+Y)d+b}\right)(2X(1-\beta)+Y) \tag{37}$$

From the expression above we see that $W_F^\beta$ is quadratic in $\beta$. By calculating the second-order derivative with respect to $\beta$ of $W_F^\beta$ we find that:

$$\frac{d^2 W_F^\beta}{d\beta^2} = \frac{16X^2a^2b^2e(b+(X+Y)d)\left(4X^2d^2(1-3\beta(1-\beta))+2XYd^2+Y^2d^2+2(X+Y)bd+b^2\right)}{(b+Yd+2\beta Xd)^3((b+2Xd+Yd-2X\beta d)^3)} \geq 0 \tag{38}$$

the second-order-derivative is positive for all $0 \leq \beta \leq 1$. Hence $W_F^\beta$ is strictly convex in $\beta$ and welfare is maximized in one of the corners. In the paper we considered a symmetric situation where all parameters in the suburbs were equal.

As a side note we can consider a non-symmetric situation where $Y_i > Y_j$. By taking the derivate with respect to $\beta$ and solving the first-order-condition for $\beta$ we get:

$$\beta^{min} = \frac{2X-(Y_i-Y_j)}{4X} < \frac{1}{2} \tag{39}$$

This is the value of $\beta$ that minimizes federal welfare given that $z_i$ and $z_j$ are optimally chosen. Since a quadratic function is symmetric in $\beta$ around its minimum, this implies that federal welfare is maximized if all traffic is directed to suburb $i$. Federal welfare is hence maximized by directing all transit traffic to the suburb with the highest level of local traffic and invest more on externality reduction there. If $Y_i = Y_j$ the federal government is indifferent between which of the suburbs to direct all traffic to.

## Formal proof of equation 35

To prove that the local governments prefer noise walls to speed bumps when the federal government is indifferent between the two measures we consider the expression for $\Delta W^{local}$ in equation (36). By inserting the optimal levels of traffic calming $z_d = \frac{ea(X+Y)}{(X+Y)d+b}$ and $z_0 = \frac{ea'(X+Y)}{b'}$ from equation (33) into equation (35) we get:

$$
\begin{aligned}
W_d^{local} &= W_d^{federal} + \left( c + \frac{d}{2} \left( \frac{ea(X+Y)}{(X+Y)d+b} \right)^2 \right) X \\
W_0^{local} &= W_0^{federal} + cX
\end{aligned}
$$

(40)

Inserting these into equation (36) we can show that:

$$
\begin{aligned}
\Delta W^{local} &= W_0^{local} - W_d^{local} = \\
\left( W_0^{federal} + cX \right) &- \left( W_d^{federal} + \left( c + \frac{d}{2} z_d^2 \right) X \right) = \\
cX - \left( c + \frac{d}{2} z_d^2 \right) X &= -\frac{d}{2} \left( \frac{ea(X+Y)}{(X+Y)d+b} \right)^2 X < 0
\end{aligned}
$$

(41)