The Inefficiency of Marginal cost pricing on roads

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The economic principle of road pricing is that a road toll should equal the marginal cost imposed by an additional user, since this will lead to efficient use of the transport facility. However, when the road is used by traffic both from the road providing region as well as by traffic from another region, the supplied road standard is likely to be too low, since the consumer surplus of the users from outside the region is not taken into account.

This can be solved by letting an authority level higher than the road supplier use taxes and earmarked transactions to raise the road standard. (In Europe we see this done in the Trans European Network). To do this the higher authority needs very detailed information about the road and the users on local level. Further raising taxes and transactions also involve costs, that can be substantial. Another problem is that transactions of this type it is hard to separate from other political interferance.

This paper analyzes how a limited toll on top of the marginal cost can serve the purpose of of solving this problem locally, without involving a higher authority.

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September 2, 2014

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However, when the road is used by traffic both from the road providing region as well as by traffic from another region, the supplied road standard is likely to be to low, since the consumer surplus of the users from outside the region is not taken into account.

This can be solved by letting an authority level higher than the road supplier use taxes and earmarked transactions to raise the road standard. (In Europe we see this done in the Trans European Network). To do this the higher authority needs very detailed information about the road and the users on local level. Further raising taxes and transactions also involve costs, that can be substantial. Another problem is that transactions of this type is hard to separate from other poitical interferance.

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1 Introduction

Road pricing is a growing issue in transport research. In Europe road pricing has developed along two different lines; tolls to control congestion in urban areas (for instance Stockholm and London) and marginal cost pricing in interurban areas (so far solely focused on heavy goods vehicles (HVF) introduced in Switzerland, Austria, Germany and others)

The main part of the road pricing literature has focused on the possibility to use tolls in order to control road congestion (for a short overview see Lindsey 2012).

For tolls not used to control congestion the economic principle is that a toll should equal the marginal cost imposed by an additional user, since this will lead to efficient use of the transport facility in question. This is also the pricing policy that is spreading in Europe for interurban roads.

However the situation is not as trivial as it sounds.

Quite often a road is not only used by traffic from the region supplying and financing the road, but also from other regions. The road supplying region will therefore not consider the welfare of all users when deciding quality and capacity of the road. Further, the users from outside the road providing region can not contribute to increase quality and capacity even if this would be in line with their preferences.

Another problem with road tolls is the substantial public opposition. Although when it comes to tolls in order to control congestion, experience show that the opposition quite often change to support after the tolling reform is introduced. Naturally this implies that there is a problem introducing road tolls, since politicians have to make decisions against the will of the majority (see for instance Schade and Baum (2007) and De Borger and Proost 2012).

The reason for a marginal cost policy even when there are users from outside the region trivial. The existing road will be efficiently used, and the region providing the road does not have to carry any costs for users from outside the region, nor can they exploit this traffic. However, this is precisely the cause of a welfare loss, since this will lead to a standard of the road that is not efficient considering all traffic.

This is not a new thought. The solution has been that a decision level higher than the region, providing the road, give earmarked financing to this region in order to raise quality. This is for instance the case with the Trans European Network (TEN). However, this means that this higher decision level must have very detailed information of all roads and their use. Further it means that political decisions such as supporting a certain region, via infrastructure investments, is hard to separate from the transaction to adjust road standard with respect to traffic from outside the region. Further it is questionable if the detour via a higher authority level is efficient. There is a large risk that a significant chunk of the resources are lost in this process. The EU Anti-corruption Report (2014) claim that corruption costs the European Economy 120 billions per year. This is an implication that it might be a good idea not to involve more levels than is strictly necessary in handling transactions, and to make these transactions as transparent as possible, both in order to save resources but also to increase acceptance for a region to contribute to a facility their members use in another region.

The purpose of this paper is to investigate how this could be done by adding a limited toll on top of the marginal cost. When a toll is added on top of the marginal cost, this means that the welfare function, of the region providing the road, indirectly will take the consumer surplus of users from the other region into account via the toll-income.

This would be a more transparent way of handling the problem. Further the apparatus to collect such a toll already exists due to the marginal cost pricing.

2

However, since traffic from different regions are not necessarily willing to pay equally much for raising road standard, the toll has to be set so that all traffic benefit. Therefore it is desirable that the principle for how to restrict tolls and the use of toll-income is made on a higher authority level that take an interest in the welfare of all involved regions.

The questions that needs to be answered is therefore what restrictions need to be set for the size of the toll and the use of the toll income.

A simple model with two regions; region A and region B is used. Region A provides a road which is used by traffic from both A and B. The two regions in the model can be interpreted as for instance communities or nations. This is a very general setting applicable to various situations, such as adjacent communities with a commuting population, or nations with highways with international traffic.

The paper analyzes and compare three different toll systems.

- Marginal cost pricing

- a toll on top on marginal cost pricing, where toll incomes can be used freely

- a toll on top of marginal cost pricing, where toll incomes are used to increase quality and capacity

In the first part of the paper it is assumed that there is no congestion. The road supplying region A sets the quality. In the second part quality is assumed to be fixed. The road suffers from congestion, and region A choose capacity.

Policy implications are given in the last section.

2 Model with no congestion, region A decides quality

In this first version of the model, it is assumed that there is no congestion. Region A is providing a road used by traffic from both region A and region B. The variable to decide for region A is with what quality to provide the road.

The traffic volumes are denoted by the strictly positive functions x^A and x^B.

The inverse demand functions are given by

$$p^i = a^i - b^i x^i$$

where the coefficients a^i and b^i are strictly positive real numbers[1].

---

[1] The reason to chose an inverse linear demand function is to simplify the further analysis.

2.1 Usage charged with marginal cost

The supplier of the road, region A, is allowed to charge the users for their marginal cost of their usage. The marginal cost is assumed either to be fixed, or to depend on the quality with which the road is provided.

The generalized user cost functions are given by

$$g^i = \alpha^i + \frac{\beta^i}{q} \text{ for i} \in \{A, B\}$$

where α^i and β^i are assumed to be strictly positive real numbers.

If the marginal cost is assumed to be fixed, this cost is included in α^i. If the marginal cost is dependent on the quality, we assume that it is inverse proportional to the quality, in which case it is included in $\frac{\beta^i}{q}$.

In equilibrium the generalized prices equals the generalized user cost, thus

$$a^i - b^i x^i = \alpha^i + \frac{\beta^i}{q} \tag{1}$$

From equation (1) the traffic volumes x^A and x^B can be determined.

$$x^i = \frac{1}{b^i}\left(a^i - \alpha^i - \frac{\beta^i}{q}\right) \text{ for i} \in \{A, B\} \tag{2}$$

The welfare of region A, providing the road, consists of the consumer surplus for users from this region $\int_0^{x^A} p^A(x)\,dx$, minus the user cost $x^i g^i$, minus the costs for providing the road with the chosen quality q. For simplicity let the cost of providing the road with quality q be q.

The welfare of region A is now given by

$$
\begin{aligned}
W^A &= \int_0^{x^A} p^A(x)\,dx - x^A g^A - q \\
&= \frac{b^A}{2}\left(x^A\right)^2 - q
\end{aligned}
$$

Remember that $p^i = g^i$ in equilibrium. The welfare of region B consists of the consumer surplus, for users from this region, minus the user cost. Thus

$$
\begin{aligned}
W^B &= \int_0^{x^B} p^B(x)\,dx - x^B g^B \\
&= \frac{b^B}{2}\left(x^B\right)^2 \tag{3}
\end{aligned}
$$

Region A will choose the quality in order to maximize its welfare. Therefore, to find the welfare-maximizing quality, the welfare function of region A needs to be analyzed.

4

$$W^A = \frac{b^A}{2}\left(x^A\right)^2 - q$$

$$= \frac{b^A}{2}\frac{1}{\left(b^A\right)^2}\left(a^A - \alpha^A - \frac{\beta^A}{q}\right)^2 - q$$

$$= \frac{1}{2b^A}\left(a^A - \alpha^A - \frac{\beta^A}{q}\right)^2 - q \tag{4}$$

The first order condition gives that the welfare optimizing toll q satisfies

$$\frac{dW^A}{dq} = \frac{1}{b^A}\left(a^A - \alpha^A - \frac{\beta^A}{q}\right)\left(\frac{\beta^A}{q^2}\right) - 1 = 0$$

$$\frac{\beta^A}{b^A}\left(a^A - \alpha^A - \frac{\beta^A}{q}\right) = q^2$$

$$\frac{\beta^A}{b^A}\left(a^A - \alpha^A\right)q - \frac{\left(\beta^A\right)^2}{b^A} = q^3$$

$$q^3 - \frac{\beta^A}{b^A}\left(a^A - \alpha^A\right)q + \frac{\left(\beta^A\right)^2}{b^A} = 0 \tag{5}$$

This is a cubic function, and it is well known that such a function has three roots of which either two or no root is complex. Even though we therefore know that this function has at least one real root it is not possible to express this solution, in the general case, without using complex numbers. This expression will therefore be of no help in the further analysis of this paper. However it is possible to determine some properties of the welfare maximizing root q^0.

By assumption x^A, x^B, W^A, W^B, and q are strictly positive. Indirectly this means that we have assumed that there is a positive root q^0 yielding $x^A, W^A > 0$.

Lemma 1 *Assume that there is a $q^0 > 0$ such that q^0 maximizes the function $W^A = \frac{1}{2b^A}\left(x^A\right)^2 - q$, with $x^A = \left(a^A - \alpha^A - \frac{\beta^A}{q}\right) > 0$. Then q^0 is the largest root of the cubic function $q^3 - \frac{\beta^A}{b^A}\left(a^A - \alpha^A\right)q + \frac{\left(\beta^A\right)^2}{b^A} = 0$.*

Proof. The first order condition for a maximum of W^A is given by equation (5) as) as above, i.e. the cubic function $q^3 - \frac{\beta^A}{b^A}\left(a^A - \alpha^A\right)q + \frac{\left(\beta^A\right)^2}{b^A} = 0$. Thus the three roots to this cubic function are the extreme points of the function W^A.

It is trivial to see that this function is continuos everywhere except in the point q=0 where the function has an incontinuity such that $\lim_{q\to 0-} W^A(q) =$

∞, and $\lim_{q \to 0^+} W^A(q) = \infty$. Further the function has two asymptotes; $\lim_{q \to \infty} \left(W^A(q) + q \right) = \frac{1}{b^A} \left(a^A - \alpha^A \right)$ and $\lim_{q \to -\infty} \left(W^A(q) + q \right) = \frac{1}{b^A} \left(a^A - \alpha^A \right)$.

Since $\lim q \to 0^- = \infty$ and $\lim_{q \to \infty} \left(W^A(q) + q \right) = \frac{1}{b^A} \left(a^A - \alpha^A \right)^2$ it follows from continuity that there is at least one local minimum where q<0. Since $\lim q \to 0^+ = \infty$, it is trivial to see that W^A is positive for q-values close to zero. Further q=$\frac{\beta^A}{a^A - \alpha^A}$ gives $W^A = -\frac{\beta^A}{a^A - \alpha^A}$ and x$^A = \left(a^A - \alpha^A - \frac{\beta^A}{q} \right) = 0$, meaning that the function W^A cross the x-axis at least once between q=0 and q=$\frac{\beta^A}{a^A - \alpha^A}$. Since we have assumed that there is a q^0>0 with x$^A > 0$ it follows from continuity that the function W^A has a local minimum in the interval q$\in \left(0, q^0 \right)$. From this we can deduce that the q^0 that maximizes the function W^A is larger than the q-values of the other two local extreme-points, thus q^0 is the largest q-vaue satisfying the first order condition given by the cubic function, meaning that q^0 is the largest root of this function. ∎

Even though we do not have the exact expression for the quality, the result above is enough to make it possible to do a comparison between the effects of marginal cost pricing and a toll on top of marginal cost pricing.

2.2 Usage charged with a toll on top of marginal cost

Now assume that region A, that provides the road, is allowed to charge a fixed toll $\tau > 0$ on top of the marginal cost. Region A does not have to invest the toll incomes in the road.

This means that the general user cost function is given by

$$g^i = \alpha^i + \frac{\beta^i}{q} + \tau$$

In equilibrium the generalized user cost equals the generalized price;

$$\alpha^i + \frac{\beta^i}{q} + \tau = a^i - b^i x^i$$

The traffic flows can now be expressed by:

$$x^i = \frac{1}{b^i} \left(a^i - \alpha^i - \frac{\beta^i}{q} - \tau \right) \text{ for } i \in \{A, B\}$$

For simplicity lets denote

$$
\begin{aligned}
a^A - \alpha^B &= A \\
a^B - \alpha^B &= B
\end{aligned}
$$

The welfare of region A is given by the consumer surplus minus the user cost plus the toll incomes minus the road provider costs.

6

$$W_\tau^A = \int_0^{x^A} p^A(x)\, dx - x^A g^A - q + \left(x^A + x^B\right)\tau$$

$$= \frac{b^A}{2}\left(x^A\right)^2 - q + \left(x^A + x^B\right)\tau$$

$$= \frac{1}{2b^A}\left(A - \frac{\beta^A}{q} - \tau\right)^2 - q + \frac{\tau}{b^A}(A + \frac{\beta^A}{q} - \tau) + \frac{\tau}{b^B}(B + \frac{\beta^B}{q} - \tau)$$

$$= \frac{1}{2b^A}\left(A - \frac{\beta^A}{q}\right)^2 - q + \frac{\tau^2}{2b^A} - \frac{2\tau}{2b^A}\left(A - \frac{\beta^A}{q}\right) + \frac{\tau}{b^A}(A + \frac{\beta^A}{q} - \tau) + \frac{\tau}{b^B}(B + \frac{\beta^B}{q} - \tau)$$

$$= \frac{1}{2b^A}\left(A - \frac{\beta^A}{q}\right)^2 - q + \frac{\tau^2}{2b^A} - \frac{\tau^2}{b^A} + \frac{\tau}{b^B}(B + \frac{\beta^B}{q} - \tau)$$

$$= \frac{1}{2b^A}\left(A - \frac{\beta^A}{q}\right)^2 - q - \frac{\tau^2}{2b^A} + \frac{\tau}{b^B}(B + \frac{\beta^B}{q} - \tau)$$

$$= \frac{1}{2b^A}\left(A - \frac{\beta^A}{q}\right)^2 - q - \frac{\tau^2}{2b^A} + \frac{\tau}{b^B}(B - \tau) + \frac{\beta^B \tau}{b^B q} \tag{6}$$

The welfare of region B is given by the consumer surplus minus the user cost

$$W_\tau^B = \int_0^{x^B} p^B(x)\, dx - x^B g^B$$

$$= \frac{b^B}{2}\left(x^B\right)^2 \tag{7}$$

$$= \frac{1}{2b^B}\left(B - \frac{\beta^B}{q} - \tau\right)^2$$

The road providing region sets the quality q to maximize its welfare. The first order condition for the welfare maximizing q is given by derivating equation (6).

$$\frac{dW_\tau^A}{dq} = \frac{1}{b^A}\left(A - \frac{\beta^A}{q}\right)\left(\frac{\beta^A}{q^2}\right) - 1 - \frac{1}{q^2}\left(\frac{\beta^B}{b^B}\right)\tau = 0 \tag{8}$$

Lets multiply (8) by q^3

$$0 = \frac{1}{b^A}\left(A - \frac{\beta^A}{q} - \tau\right)\left(\frac{\beta^A}{q^2}\right) - 1 - \frac{1}{q^2}\left(\frac{\beta^B}{b^B}\right)\tau$$

$$= \frac{\beta^A}{b^A}\left(Aq - \beta^A - \tau q\right) - q^3 - \left(\frac{\beta^B}{b^B}\right)\tau q$$

$$= \left[q_\tau^3 - \frac{\beta^A}{b^A}Aq_\tau + \frac{\left(\beta^A\right)^2}{b^A}\right] - \left(\frac{\beta^B}{b^B}\right)\tau q_\tau \qquad (9)$$

Let q^τ be the quality maximizing W_τ^A. Looking at function W_τ^A and its condition for extreme points given by the cubic function (9), it is easy to see that we can use the same reasoning as in Lemma 1. Thus q^t is the largest root to equation (9).

Theorem 2 *The quality will increase when A is allowed to charge a toll τ on top of the marginal cost.*

Proof. In order to show that $q^\tau > q^0$ lets compare the first order conditions for maximizing the welfare of region A with and without the toll τ (equation (5) and (9) respectively).

$$q^3 - \frac{\beta^A}{b^A}Aq + \frac{\left(\beta^A\right)^2}{b^A} = 0$$

$$q^3 - \frac{\beta^A}{b^A}Aq + \frac{\left(\beta^A\right)^2}{b^A} = \left(\frac{\beta^B}{b^B}\right)\tau q$$

By assumption q^0 and q^τ are larger than zero, and we know from Lemma 1 that these are the largest roots to equation (5) and (9) respectively. The difference between equation (5) and (9) is a positive term $\left(\frac{\beta^B}{b^B}\right)\tau q^\tau$. From this it follows that q^τ satisfying equation (5) has to be larger than q^0 satisfying equation (9). Hence $q^\tau > q^0$. ∎

The welfare of region A can be written as

$$W_\tau^A = \frac{1}{2b^A}\left(A - \frac{\beta^A}{q}\right)^2 - q - \frac{\tau^2}{2b^A} + \frac{1}{b^B}(B - \frac{\beta^B}{q} - \tau)\tau +$$

$$= W^A - \frac{\tau^2}{2b^A} + \frac{1}{b^B}(x^B)\tau$$

When the welfare function of region A is written on this form, it is trivial to see that in the case of only local traffic ($x^B = 0$), a toll will reduce the welfare of

A, i.e. the region is better off if the local government pays all costs for the road rather than the travellers contributing to these costs. However, when there is traffic from region B also contributing to the costs for the road the situation is different. It is trivial to see that when the traffic flow from B is large enough, A will be better off with a toll then without.

The next question to ask is therefore whether it can be beneficial for both region A and region B to introduce a toll.

Theorem 3 *When the quality q_0 satisfies the equation*

$$q_0^3 < \frac{1}{2}\left[\frac{\left(\beta^A\right)^2}{b^A} + \frac{\left(\beta^B\right)^2}{b^B}\right]$$

it is possible to set a toll τ, resulting in a new quality q_τ, such that both A and B benefits from introducing this toll even though A can use the profit free.

Proof. From equation (9) $q^3 - \frac{\beta^A}{b^A}\left(a^A - \alpha^A\right)q + \frac{\left(\beta^A\right)^2}{b^A} - \left(\frac{\beta^B}{b^B}\right)\tau q = 0$ we can see how q depends on the toll τ. Lets derivate this expression with respect to the toll τ, while viewing the quality as a function of the toll, i.e. $q(\tau)$.

$$3q^2\frac{dq}{d\tau} - \frac{\beta^A}{b^A}A\frac{dq}{d\tau} - \left(\frac{\beta^B}{b^B}\right)q - \left(\frac{\beta^B}{b^B}\right)\tau\frac{dq}{d\tau} = 0$$

$$\frac{dq}{d\tau}\left[3q^2 - \frac{\beta^A}{b^A}A - \left(\frac{\beta^B}{b^B}\right)\tau\right] = \left(\frac{\beta^B}{b^B}\right)q$$

$$\frac{dq}{d\tau}\left[2q^2 + \frac{1}{q}\left[q^3 - \frac{\beta^A}{b^A}Aq - \left(\frac{\beta^B}{b^B}\right)\tau q\right]\right] = \left(\frac{\beta^B}{b^B}\right)q$$

$$\frac{dq}{d\tau}\left[2q^3 - \frac{\left(\beta^A\right)^2}{b^A}\right] = \left(\frac{\beta^B}{b^B}\right)q^2$$

$$\frac{dq}{d\tau} = \frac{q^2\beta^B}{b^B\left(2q^3 - \frac{\left(\beta^A\right)^2}{b^A}\right)} \quad (10)$$

Since $q_\tau > q_0$ we know that $\frac{dq}{d\tau} > 0$ in the point $q_\tau = q_0, \tau = 0$ this means that $2q_0^3 - \frac{\left(\beta^A\right)^2}{b^A} > 0$. Further, since $q_\tau > q_0$ we can deduce that $2q_\tau^3 - \frac{\left(\beta^A\right)^2}{b^A} > 0$. Hence $\frac{dq}{d\tau} > 0$ everywhere in the interval (q_0, ∞).

Now lets derivate equation (10) with respect to τ.

$$
\frac{d^2q}{d\tau^2} = \frac{\beta^B}{b^B} 2q \frac{dq}{d\tau} \frac{1}{2q^3 - \frac{(\beta^A)^2}{b^A}} - \frac{\beta^B}{b^B} q^2 \frac{1}{\left(2q^3 - \frac{(\beta^A)^2}{b^A}\right)^2} 6q^2 \frac{dq}{d\tau}
$$

$$
= \frac{dq}{d\tau} \frac{\beta^B}{b^B} \frac{2q}{2q^3 - \frac{(\beta^A)^2}{b^A}} \left[1 - \frac{3q^3}{2q^3 - \frac{(\beta^A)^2}{b^A}}\right]
$$

Since $3q^3 > 2q^3 - \frac{(\beta^A)^2}{b^A} > 0$ it is obvious that $\frac{d^2q}{d\tau^2} < 0$ in the interval $q \in (q_0, \infty)$. This means that even if the quality increase when the toll τ is raised, the proportion between the raise in toll and the quality change becomes less and less beneficial for B the larger the toll is. For this reason it is enough to check if it is possible that a small toll is beneficial for B (since if this is not the case neither will a large change).

It is trivial to see from the welfare function of region B with and without toll (equation (3) and (7))

$$
W_0^B = \frac{b^B}{2} \left(x_0^B\right)^2
$$

$$
W_\tau^B = \frac{b^B}{2} \left(x_\tau^B\right)^2
$$

that B benefits from the toll if $x_\tau^B > x_0^B$. Thus

$$
\frac{1}{b^B} \left(B - \frac{\beta^B}{q_\tau} - \tau\right) > \frac{1}{b^B} \left(B - \frac{\beta^B}{q_0}\right)
$$

$$
\beta^B \left(\frac{1}{q_0} - \frac{1}{q_\tau}\right) > \tau \tag{11}
$$

Using the implicit function theorem we can now derivate the expression (11) in the point $\tau = 0$, with respect to τ while viewing q_τ as a function of τ.

$$
\frac{\beta^B}{q_\tau^2} \frac{dq}{d\tau} > 1
$$

$$
\frac{dq}{d\tau} > \frac{q_\tau^2}{\beta^B} = \frac{q_0^2}{\beta^B}
$$

This means that region B benefits from the toll τ if the introduction of the toll results in a raise in q that is larger than $\frac{q_0^2}{\beta^B}$. Note that $q_0 = q_\tau$ in the point $\tau = 0$.

Now using equation (10) we have that

$$\frac{q_0^2 \beta^B}{b^B \left(2q_0^3 - \frac{\left(\beta^A\right)^2}{b^A}\right)} > \frac{q_0^2}{\beta^B}$$

$$\frac{\left(\beta^B\right)^2}{b^B} > 2q_0^3 - \frac{\left(\beta^A\right)^2}{b^A}$$

$$q_0^3 < \frac{1}{2}\left[\frac{\left(\beta^A\right)^2}{b^A} + \frac{\left(\beta^B\right)^2}{b^B}\right] \tag{12}$$

Thus if (12) holds, region B benefits from the toll.

We now need to check criteria for when A benefits from a small toll, i.e. when , i.e. when $\lim_{\tau \to 0}$. This means that we want to check when $\frac{dW_\tau^A}{d\tau} > 0$ in the point $\tau = 0$.

$$\lim_{\tau \to 0} \frac{W_\tau^A - W_0^A}{\tau} = \frac{dW_\tau^A(0)}{d\tau} > 0$$

Therefore lets derivate W_τ^A with respect to τ

$$W_\tau^A = \frac{1}{2b^A}\left(A - \frac{\beta^A}{q_\tau} - \tau\right)^2 - q_\tau + \frac{\tau}{b^B}\left(B - \frac{\beta^B}{q_\tau} - \tau\right) + \frac{\tau}{b^A}\left(A - \frac{\beta^A}{q_\tau} - \tau\right)$$

$$\frac{dW_\tau^A}{d\tau} = \frac{1}{b^A}\left(A - \frac{\beta^A}{q_\tau} - \tau\right)\left(\frac{\beta^A}{q_\tau^2}\frac{dq}{d\tau} - 1\right) - \frac{dq}{d\tau} + \frac{1}{b^B}\left(B - \frac{\beta^B}{q_\tau} - \tau\right) + \frac{1}{b^A}\left(A - \frac{\beta^A}{q_\tau} - \tau\right)$$

$$+ \tau\left[\frac{\beta^B}{q_\tau^2}\frac{dq}{d\tau} - 1 + \frac{\beta^A}{q_\tau^2}\frac{dq}{d\tau} - 1\right]$$

$$= \frac{1}{b^A}\left(A - \frac{\beta^A}{q_\tau} - \tau\right)\left(\frac{\beta^A}{q_\tau^2}\frac{dq}{d\tau}\right) - \frac{dq}{d\tau} + \frac{1}{b^B}\left(B - \frac{\beta^B}{q_\tau} - \tau\right) + \tau\left[\frac{\beta^B}{q_\tau^2}\frac{dq}{d\tau} - 1 + \frac{\beta^A}{q_\tau^2}\frac{dq}{d\tau} - 1\right]$$

Now lets use that $\tau = 0$.

$$\frac{dW_\tau^A(0)}{d\tau} = \frac{1}{b^A}\left(A - \frac{\beta^A}{q_\tau}\right)\left(\frac{\beta^A}{q_\tau^2}\frac{dq}{d\tau}\right) - \frac{dq}{d\tau} + \frac{1}{b^B}\left(B - \frac{\beta^B}{q_\tau}\right)$$

$$= \frac{dq}{d\tau}\left[\frac{1}{b^A}\left(A - \frac{\beta^A}{q_0}\right)\left(\frac{\beta^A}{q_0^2}\right) - 1\right] + \frac{1}{b^B}\left(B - \frac{\beta^B}{q_\tau}\right)$$

$$= -\frac{dq}{d\tau}\frac{1}{q_0^3}\left[q_0^3 - \frac{\beta^A}{b^A}Aq_0 + \frac{\left(\beta^A\right)^2}{b^A}\right] + \frac{1}{b^B}\left(B - \frac{\beta^B}{q_\tau}\right)$$

11

From equation (5) we know that $q_0^3 - \frac{\beta^A}{b^A} A q_0 + \frac{(\beta^A)^2}{b^A} = 0$. Thus

$$\frac{dW_\tau^A(0)}{d\tau} = \frac{1}{b^B}\left(B - \frac{\beta^B}{q_\tau}\right) > 0$$

Since $\frac{1}{b^B}\left(B - \frac{\beta^B}{q_0}\right) > 0$ is always true a small toll will always be beneficial for A.

Hence both region A and B will benefit from a toll satisfying $q_0^3 < \frac{1}{2}\left[\frac{(\beta^A)^2}{b^A} + \frac{(\beta^B)^2}{b^B}\right]$.

■

This shows that a small toll on top of the marginal cost will always increase the quality, even though region A does not have to invest the toll incomes in the road.. Further, if quality is enough important for region B (a large β^B means that the users from region B are very sensitive to quality changes) such a toll will be beneficial for both regions.

2.3 Usage charged with marginal cost plus a toll used to increase quality

Lets now consider a situation where region A is allowed to charge a toll τ, but has to use the toll incomes to increase the quality from q^0 (the quality in the case of marginal cost pricing) to the quality $q^0 + \tau\left(x^A + x^B\right)$. Further for simplicity lets assume that $x^A = x^B = x$.

Thus the inverse demand function is given by

$$p = a - bx$$

and the generalized user cost is given by

$$g = \alpha + \frac{\beta}{q^0 + 2\tau x} + \tau$$

In equilibrium the generalized price equals the generalized user cost.

$$A - \tau = bx + \frac{\beta}{q^0 + 2\tau x}{}'$$
$$x_\tau = \frac{1}{b}\left(A - \tau - \frac{\beta}{q + 2\tau x^\tau}\right)$$

The welfare of region A and B are given by the generalized user cost is now given by

$$g = \alpha + \frac{2\beta x}{q} + \tau$$

Thus in equilibrium we have that

$$a - bx = \alpha + \frac{2\beta x}{q} + \tau$$

$$A - \tau = x(b + \frac{2\beta}{q})$$

Which gives the traffic flows

$$x = \frac{A - \tau}{b + \frac{2\beta}{q}}$$

$$= \frac{(A - \tau) q}{bq + 2\beta}$$

Theorem 4 *If the toll $\tau < \frac{\beta}{2q^0}$ both region A and region B will benefit from the toll.*

Proof. It is trivial to see from the welfare functions that both region A and region B will benefit from the toll if $x^\tau > x^0$.

Lets assume that $x^\tau > x^0$. *Thus*

$$
\begin{aligned}
x^\tau &> x^0 \\
\frac{1}{b}\left(A - \tau - \frac{\beta}{q^0 + 2\tau x^\tau}\right) &> \frac{1}{b}\left(A - \frac{\beta}{q^0}\right) \\
-\tau - \frac{\beta}{q^0 + 2\tau x^\tau} &> -\frac{\beta}{q^0} \\
\tau + \frac{\beta}{q^0 + 2\tau x^\tau} &< \frac{\beta}{q^0} \\
\tau + \frac{\beta}{q^0 + 2\tau x^\tau} &< \tau + \frac{\beta}{q^0 + 2\tau x^0} < \frac{\beta}{q^0} \\
\tau + \frac{\beta}{q^0 + 2\tau\left(\frac{1}{b}\left(A - \frac{\beta}{q^0}\right)\right)} &< \frac{\beta}{q^0} \\
\tau\left(q^0 + \frac{2\tau}{b}\left(A - \frac{\beta}{q^0}\right)\right)q^0 + \beta q^0 &< \beta\left(q^0 + \frac{2\tau}{b}\left(A - \frac{\beta}{q^0}\right)\right) \\
q^0\left(q^0 + \frac{2\tau}{b}\left(A - \frac{\beta}{q^0}\right)\right) &< \frac{2\beta}{b}\left(A - \frac{\beta}{q^0}\right) \\
\left(q^0\right)^2 + \frac{2\tau q^0}{b}\left(A - \frac{\beta}{q^0}\right) &< \frac{2\beta}{b}\left(A - \frac{\beta}{q^0}\right) \\
\frac{2\tau q^0}{b} &< \frac{\frac{2\beta}{b}\left(A - \frac{\beta}{q^0}\right) - \left(q^0\right)^2}{A - \frac{\beta}{q^0}} \\
\tau &< \frac{\beta}{q^0} - \frac{bq^0}{2\left(A - \frac{\beta}{q^0}\right)} \\
\tau &< \frac{\beta}{q^0} - \frac{b\left(q^0\right)^2}{2\left(Aq^0 - \beta\right)} \qquad (13)
\end{aligned}
$$

From equation (5) we have that the welfare maximizing quality in the case of only marginal cost pricing satisfies

$$
\begin{aligned}
\left(q^0\right)^3 - \frac{\beta}{b}Aq^0 + \frac{\beta^2}{b} &= 0 \\
\left(q^0\right)^2 &= \frac{\beta}{b}A - \frac{\beta^2}{bq^0} \\
\left(q^0\right)^2 &= \frac{\beta}{b}\left(A - \frac{\beta}{q^0}\right) \qquad (14)
\end{aligned}
$$

14

When inserting (14) in (13) above we have that

$$
\begin{aligned}
\tau &< \frac{\beta}{q^0} - \frac{b\frac{\beta}{b}\left(A - \frac{\beta}{q^0}\right)}{2\left(Aq^0 - \beta\right)} \\
\tau &< \frac{\beta 2\left(Aq^0 - \beta\right) - \beta(Aq^0 - \beta)}{2q^0\left(Aq^0 - \beta\right)} \\
\tau &< \frac{\beta\left(Aq^0 - \beta\right)}{2q^0\left(Aq^0 - \beta\right)} \\
\tau &< \frac{\beta}{2q^0}
\end{aligned}
$$

Thus if the toll τ is smaller than $\frac{\beta}{2q^0}$ it is true that $x^\tau > x^0$, and both region A and B benefit from introducing the toll. ∎

Obviously quality will increase when this toll is introduced. If the toll is not to large both regions will benefit from this toll reform.

3 Model with congestion, region A decides capacity

In this version of the model we assume that the road in question is congested. The variable for the road providing region A is the capacity. Quality is assumed to be fixed.

The traffic volumes are denoted by the strictly positive functions x^A and x^B.

Adding congestion makes the model very much more complicated to analyze. Therefore lets make the simplification that the traffic flows are equal, thus $x^A = x^B = x$.

The inverse demand functions are given by

$$p = a - bx$$

where the coefficients a and b are strictly positive real numbers. The road is assumed to be congested. Further the providing region A is allowed to charge the users for their marginal cost. The congestion cost is not included in the marginal cost since it is not a cost that falls upon the providing region but on the users.

The congestion cost is inverse proportional to the capacity q.

3.1 Usage charged with marginal cost

The generalized user cost function is given by the value of time times the inverse of the capacity per vehicle.

$$g = \alpha + \frac{2\beta x}{q}$$

For simplicity lets denote

$$A = a - \alpha$$

In equilibrium the generalized prices equals the generalized user cost, thus

$$a - bx = \alpha + \frac{2\beta x}{q}$$

$$A = x(b + \frac{2\beta}{q})$$

$$x = \frac{A}{b + \frac{2\beta}{q}}$$

The welfare of region A (providing the road) consists of the consumer surplus of the users from region A $\int_0^x p(x)\, dx$, minus the user cost xg, minus the costs for providing the road with the chosen capacity q. For simplicity let the cost of providing the road with capacity q be q. The welfare of region A is therefore given by

$$W^A = \int_0^x p(x)\, dx - xg - q$$

$$= \frac{b}{2}x^2 - q \tag{15}$$

and the welfare of region B is analogously given by

$$W^B = \int_0^x p(x)\, dx - xg$$

$$= \frac{b}{2}x^2 \tag{16}$$

The providing region will choose the capacity in order to maximize the welfare of the region. The first order condition for a maximum is given by

$$W_0^A = \frac{b}{2}x^2 - q$$

$$\frac{dW_0^A}{dq} = b\frac{Aq}{bq + 2\beta}\left(\frac{A(bq + 2\beta) - Abq}{(bq + 2\beta)^2}\right) - 1$$

$$bA^2 q 2\beta - (bq + 2\beta)^3 = 0$$

$$2A^2 b\beta q - (bq + 2\beta)\left(b^2 q^2 + 4\beta^2 + 4b\beta q\right) = 0$$

$$2A^2 b\beta q - \left(b^3 q^3 + 4\beta^2 bq + 2\beta b^2 q^2 + 8\beta^3 + 4b^2\beta q^2 + 8b\beta^2 q\right) = 0$$

$$b^3 q^3 + 6\beta b^2 q^2 + q\left(12b\beta^2 - 2A^2 b\beta\right) + 8\beta^3 = 0 \tag{17}$$

We can now reason analogously to lemma 1 and deduce that the capacity that maximizes the welfare W_0^A is the largest root to equation (17). Lets denote this root by q^0.

3.2 usage charged with marginal cost plus toll with free use

Now assume that region A is allowed to charge a toll τ. The generalized user cost is now given by

$$g = \alpha + \frac{2\beta x}{q} + \tau$$

Thus in equilibrium we have that

$$a - bx = \alpha + \frac{2\beta x}{q} + \tau$$

$$A - \tau = x(b + \frac{2\beta}{q})$$

Which gives the traffic flows

$$x = \frac{A - \tau}{b + \frac{2\beta}{q}}$$

$$= \frac{(A - \tau)\,q}{bq + 2\beta}$$

Region A will set the capacity in order to maximize its welfare $W_\tau^A = \frac{b}{2}x^2 - q + 2\tau x$. The first order condition for a maximum is given by

$$W_\tau^A = \frac{b}{2}x^2 - q + 2\tau x$$

$$\frac{dW_\tau^A}{dq} = bx\frac{dx}{dq} - 1 + 2\tau\frac{dx}{dq} = 0$$

$$0 = \left(\frac{2A\beta}{(bq+2\beta)^2} - \frac{2\beta\tau}{(bq+2\beta)^2}\right)\left(\frac{Abq}{bq+2\beta} - \frac{b\tau q}{bq+2\beta} - 2\tau\right) - 1$$

$$= \frac{1}{(bq+2\beta)^3}\left(2A^2 b\beta q - 2Ab\beta\tau q - 2Ab\beta\tau q + 2b\beta\tau^2 q - 4\beta\tau(bq+2\beta)(A-\tau)\right) - 1$$

$$= \left(2A^2 b\beta q - 4Ab\beta\tau q + 2b\beta\tau^2 q - 4\beta\tau(Abq - b\tau q + 2A\beta - 2\beta\tau)\right) - (bq+2\beta)^3$$

$$= 2A^2 b\beta q - 4Ab\beta\tau q + 2b\beta\tau^2 q - 4\beta\tau Abq + 4\beta b\tau^2 q - 8A\beta^2\tau + 8\beta^2\tau^2 - (bq+2\beta)^3$$

$$= 2A^2 b\beta q - 8Ab\beta\tau q + 6b\beta\tau^2 q - 8A\beta^2\tau + 8\beta^2\tau^2 - b^3 q^3 - 124bb\beta^2 q - 6b^2\beta q^2 - 8\beta^3$$

$$b^3 q^3 + 6b^2\beta q^2 + q\left(12b\beta^2 - 2A^2 b\beta\right) + 8\beta^3 = -8Ab\beta\tau q + 6b\beta\tau^2 q - 8A\beta^2\tau + 8\beta^2\tau^2$$

$$b^3 q^3 + 6b^2\beta q^2 + q\left(12b\beta^2 - 2A^2 b\beta\right) + 8\beta^3 = 2\beta\tau - 4Abq + 3b\tau q - 4A\beta + 4\beta\tau$$

$$b^3 q^3 + 6b^2\beta q^2 + q\left(12b\beta^2 - 2A^2 b\beta\right) + 8\beta^3 = 2\beta\tau\left(-4(A-\tau)(bq+\beta\tau) - b\tau q\right) \quad (18)$$

Reasoning as in lemma 1 we can deduce that the maximum welfare is given by the largest root q^τ to equation (18).

Theorem 5 *The capacity decided by region A will decrease when A is allowed to charge a toll τ*

Proof. In order to show that $q^\tau < q^0$ lets compare the first order conditions for maximizing the welfare of region A with and without the toll τ.(equation (17) and (18) respectively)

$$b^3 q^3 + 6\beta b^2 q^2 + q\left(12b\beta^2 - 2A^2 b\beta\right) + 8\beta^3 = 0$$
$$b^3 q^3 + 6b^2 \beta q^2 + q\left(12b\beta^2 - 2A^2 b\beta\right) + 8\beta^3 = 2\beta\tau\left(-4\left(A - \tau\right)\left(bq + \beta\tau\right) - b\tau q\right)$$

Since $(A - \tau)$ is positive we know that the expression on the right side of equation (18) is negative. By assumption q^0 and q^τ are larger than zero, and we know from Lemma 1 that these are the largest roots to equation (17) and (18) respectively. The difference between equation (17) and (18) a negative term . From this it follows that q^τ has to be smaller than q^0. Hence $q^\tau < q^0$. ∎

This result might seem counter intuitive. However this result can be explained by the fact that a toll will increase the cost for travelling also means that the demand for travelling is reduced thus reducing the need for capacity.

This result also makes it clear that region B will never benefit from such a toll. For region B to benefit from the toll it would have to be true that

$$
\begin{aligned}
x_\tau &> x_0 \\
\frac{A - \tau}{b + \frac{2\beta}{q_\tau}} &> \frac{A}{b + \frac{2\beta}{q_0}} \\
(A - \tau)\left(b + \frac{2\beta}{q_0}\right) &> A\left(b + \frac{2\beta}{q_\tau}\right) \\
\frac{2A\beta}{q_0} - \tau b - \frac{2\tau\beta}{q_0} &> \frac{2A\beta}{q_\tau}
\end{aligned}
$$

but since we know that $q_\tau < q_0$ this can never be the case.

3.2.1 toll used to increase capacity

Now lets assume that region A is allowed to charge a toll τ on top of the marginal cost given that the income from the toll are used to increase the capacity. The generalized user cost is now given by

$$g = \alpha + \frac{2\beta x}{q + 2x\tau} + \tau$$

Thus in equilibrium we have that

$$
\begin{aligned}
a - bx &= \alpha + \frac{2\beta x}{q + 2x\tau} + \tau \\
A - \tau &= x\left(b + \frac{2\beta}{q + 2x\tau}\right)
\end{aligned}
$$

18

Which gives that

$$x_{toll} = \frac{A - \tau}{b + \frac{2\beta}{q+2x\tau}}$$

The welfare function of A and B are given by

$$
\begin{aligned}
W^A &= \int_0^{x_{toll}} p(x)\,dx - x_{toll}g - q \\
&= \frac{b}{2}x_{toll}^2 - q
\end{aligned}
$$

$$
\begin{aligned}
W^B &= \int_0^{x_{toll}} p(x)\,dx - x_{toll}g \\
&= \frac{b}{2}x_{toll}^2
\end{aligned}
$$

Since the welfare of A and B without toll is given by equation (15) and (16) respectively

$$
\begin{aligned}
W^A &= \int_0^{x_0} p(x)\,dx - x_0 g - q \\
&= \frac{b}{2}x_0^2 - q
\end{aligned}
$$

$$
\begin{aligned}
W^B &= \int_0^{x_{0l}} p(x)\,dx - x_0 g \\
&= \frac{b}{2}x_0^2
\end{aligned}
$$

it is trivial to see that both region A and region B benefits from the toll if $x_{toll} > x_0$.

Theorem 6 *If the toll $\tau < \frac{4A^2\beta - (bq+2\beta)^2}{2A(2\beta+bq)}$ both region A and B benefits from a toll given that the toll incomes are used to increase the capacity*

Proof. We want to show the criteria for when $x_{toll} > x_0$. Thus

$$
\begin{aligned}
\frac{A-\tau}{b + \frac{2\beta}{q+2x_{toll}\tau}} &> \frac{A}{b + \frac{2\beta}{q}} \\
(A-\tau)\left(b + \frac{2\beta}{q}\right) &> A\left(b + \frac{2\beta}{q+2x_{toll}\tau}\right) \\
\frac{2\beta A}{q} - \tau b - \frac{2\tau\beta}{q} &> \frac{2A\beta}{q+2x_{toll}\tau}
\end{aligned}
$$

Since we have assumed that $x_{toll} > x_0$ we know that if

$$\frac{2\beta A}{q} - \tau b - \frac{2\tau\beta}{q} > \frac{2A\beta}{q+2x_0\tau} > \frac{2A\beta}{q+2x_{toll}\tau}$$

19

$$\frac{2\beta A}{q} - \tau b - \frac{2\tau \beta}{q} > \frac{2A\beta}{q + 2x_0\tau}$$

$$\frac{2\beta A}{q} - \frac{\tau bq}{q} - \frac{2\tau \beta}{q} > \frac{2A\beta}{q + 2\frac{A}{b+\frac{2\beta}{q}}\tau}$$

$$(2A\beta - \tau bq - 2\tau \beta)\left(q + 2\frac{A}{b+\frac{2\beta}{q}}\tau\right) > 2A\beta q$$

$$4\frac{A^2\beta\tau}{b+\frac{2\beta}{q}} - \tau bq^2 - 2\frac{Ab\tau^2 q}{b+\frac{2\beta}{q}} - 2\tau\beta q - 4\frac{A\beta\tau^2}{b+\frac{2\beta}{q}} > 0$$

$$4A^2\beta - bq^2\left(b + \frac{2\beta}{q}\right) - 2Ab\tau q - 2\beta q\left(b + \frac{2\beta}{q}\right) - 4A\beta\tau > 0$$

$$4A^2\beta - b^2q^2 - 2b\beta q - 2b\beta q - 4\beta^2 > 4A\beta\tau + 2Ab\tau q$$

$$4A^2\beta - (bq + 2\beta)^2 > \tau 2A(2\beta + bq)$$

$$\frac{4A^2\beta - (bq + 2\beta)^2}{2A(2\beta + bq)} > \tau$$

For this to be possible we need to check that $4A^2\beta - (bq + 2\beta)^2 > 0$. From equation (17) we can deduce that

$$(bq + 2\beta)^3 = 2A^2 b\beta q$$

Lets multiply $4A^2\beta - (bq + 2\beta)^2 > 0$ with $(bq + 2\beta)$ thus

$$
\begin{aligned}
0 &< 4A^2\beta(bq + 2\beta) - (bq + 2\beta)^3 \\
&= 4A^2\beta(bq + 2\beta) - 2A^2 b\beta q \\
&= 2A^2 b\beta q + 8A^2\beta^2
\end{aligned}
$$

Since this is always true we have that

$$\frac{4A^2\beta - (bq + 2\beta)^2}{2A(2\beta + bq)} > \tau > 0$$

■

4 Policy implications

For tolls not used to control congestion the economic principle is that a toll should equal the marginal cost imposed by an additional user, since this will lead to efficient use of the transport facility in question. This is also the pricing policy that is spreading in Europe for interurban roads.

Quite often a road is not only used by the region supplying and financing the road, but also by traffic from other regions. The region supplying the road,

will not consider the welfare of users from other regions, when deciding quality and capacity of the road.

The solution has been that a decision level, higher than the region providing the road, give earmarked financing to this region in order to raise quality. However, this means that this higher decision level must have very detailed information of all roads and their use. Further it means that political decisions such as supporting a certain region, via infrastructure investments, is hard to separate from the adjustment of quality and capacity due to users from outside the region providing the road. Further the EU Anti-corruption Report (2014) claim that corruption costs the European Economy 120 billions per year. This is an implication that it might be a good idea not to involve more levels than is strictly necessary in handling transactions, and to make these transactions as transparent as possible, in order to save resources, but also to make regions more willing to accept paying for a facility their members use in a another region.

A more transparent way to arrange this is to use a toll on top of the marginal cost, even if transactions is more efficient in a model without transaction costs and corruption. Further the apparatus to collect such a toll already exist in order to handle the marginal cost pricing.

The purpose of this paper is to investigate how this could be done by adding a limited toll on top of the marginal cost. When a toll is added on top of the marginal cost, this means that the welfare function, of the region providing the road, indirectly will take the consumer surplus of users from the other region into account via the toll-income.

However, since traffic from different regions are not necessarily willing to pay equally much for raising road standard, the toll has to be set so that all traffic benefit. Therefore it is desirable that the principle for how to restrict tolls and the use of toll-income is made on a higher authority level that take an interest in the welfare of all involved regions.

This paper has used a simple model with two regions, A and B, in order to analyze how such a toll affects quality, capacity and welfare levels, and what restrictions need to be set for the toll and the use of the toll-income

In the case of a non-congested road a toll top of the marginal cost will always lead to a raise in quality even if no restriction is put on how to use the toll-income. Moreover, if quality is very important to the users, such a toll can be beneficial for both regions.

However, in general the use of toll incomes need to be restricted to investments of the road, in order for both regions to benefit from the toll. Further, since the traffic demand for he two regions are likely to be different the toll also need to be restricted in order to make certain that both regions really benefit from paying the toll to increase quality or capacity. Such an upper level of the toll is given in the paper.

The policy implication of the paper is that a correctly set toll, on top of the marginal cost, can serve the purpose of adjusting the road standard with respect to users from outside the region. This is a simple way to avoid involving

a higher authority.[2]

There is an important questions that this paper does not answer, where further research is needed.

Is it cost efficient to finanse the discussed raise in road standard with a toll rather than with taxes and transactions? The author finds it likely that the transaction costs and lack of transparency by involving a higher decision level motivates delegating the adjustments of the road standard by tolling. However to answer this question real data would need to be analyzed.

5 References

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[2] The analyzis of the paper was complicated since it deals with the general case. However, when solving this kind of equations for specific cases there are very straight forward algorithms to be used.