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1. Introduction

Governments have a large toolbox to choose from when they want to control emissions of harmful substances.¹ In some cases the instrumental choice has no or only small efficiency implications. For instance, if equipped with complete information the planner would be able to induce the efficient emission level in a cost-effective way with any relevant instrument. In a world where abatement costs are uncertain this is no longer the case. Instruments utilizing the price mechanism (such as taxes and cap-and-trade) may still find a cost-effective allocation of abatement efforts but will in general miss the efficient emission volume. It is well-known that the optimal cap becomes too low (high) when abatement costs turn out to be higher (lower) than expected while the optimal tax induces too much (little) emissions. Weitzman (1974) shows that a cap-and-trade system yields lower expected social cost (environmental cost *plus* abatement costs) when the marginal abatement benefit function is steeper than the marginal abatement cost function, and *vice versa*.²

Much effort has been devoted to find instruments that further reduce expected social costs. Roberts and Spence (1976) show that a cap-and-trade system combined with a finite penalty of non-compliant behaviour and an abatement subsidy outperforms both the emission tax and the pure cap-and-trade system.³ However, such a full-fledged hybrid policy remains to come into systematic practical use.⁴ Pizer (1999, 2002) shows that adding only a price ceiling (a so-called safety valve) to a cap-and-trade system is sufficient to substantially reduce expected social costs. Another way is to index the emission cap to some variable that is correlated with the cost-shock (Quirion, 2005; Newell and Pizer, 2008).⁵ See also Webster,

¹ See *e.g.* Bohm and Russel (1985) and Sterner (2003) for reviews of environmental policy instruments.

² This is an important insight that also holds for stock externalities in the sense that a relative steep marginal abatement cost function increases the likelihood for the emission tax to be the preferred instrument (Hoel and Karp, 2002; Newell and Pizer, 2003). Of course, the optimal choice of policy instrument is also influenced by factors such as implementation costs and transaction costs (Stavins, 1995) and to what extent regulation rents are left in private hands (see *e.g.* Fullerton and Metcalf, 2002).

³ See also Fell, Burtraw, Morgenstern, Palmer and Preonas (2012) for a comparison of soft and hard versions of such a policy in the context of a stochastic dynamic model.

⁴ This seems to be the case also for the sliding controls studied in Yohe (1981).

⁵ Indexed emission caps/regulations have been around for a long time. For instance, the kind of green electricity certificate systems we find in Norway, the UK and Sweden express the target (production of green electricity) as a fraction of total electricity consumption. In addition, national emission quotas that are indexed to countries'

Sue Wing and Jakobovits (2010) for a comparison between indexed caps and safety valve. Mandell (2008) shows that the regulator may reduce expected social cost by using a mix of an emission tax and a cap-and-trade system. Many countries use such policy mixes. For instance, the EU has a cap-and-trade system covering carbon emissions from energy intensive industries (the EU ETS) while other emitters (*e.g.*, transportation, households and lighter industries) are subject to carbon or fossil fuel taxes. It has been suggested to index these taxes to GDP (SGOR, 2008) or to the EU ETS price (COM, 2011). It is easy to think of cases where such an indexation would enhance efficiency. However, it is also easy to think of the opposite case. To the best of our knowledge, formal analyses of the *pros* and *cons* of such an index policy are absent.

The purpose of this paper is to fill some of this gap. We study the properties of an environmental policy that (a) tax some emitters and let the others be subject to a cap-and-trade system and (b) index the tax level to the permit price. In line with most of the literature on optimal instrumental choice under uncertainty we conduct the analysis under the assumption of linear marginal cost and benefit schedules. Many of the calculations below have been made with Mathematica (Wolfram, 2010).

The rest of the paper is organized as follows. Section 2 outlines the model and recapitulates some earlier results. Section 3 presents the indexed policy and identifies the conditions under which it reduces expected social cost relatively to an emission tax, a cap-and-trade system, and the optimal combination thereof, respectively. Section 4 concludes.

2. The Model

Consider a government that wants to control the emissions of some pollutant for which the environmental cost depends only on the aggregate emission level. Abatement costs are taken to be uncertain at the point in time when the government chooses control levels. Thus, the government will in general not be able to equate marginal abatement benefits (*MABs*) with marginal abatement costs (*MACs*).⁶ Throughout the analysis we assume a government that wants to minimize expected social costs (= the sum of expected abatement costs and expected

GDP or population levels have for long been discussed as a way of attracting developing countries to a climate treaty, see Baumert *et al.* (1999), Lutter (2000), and Ellerman and Sue Wing (2003).

⁶ Also the *MAB* function may be uncertain from the regulator's perspective. However, to keep the exposition simple and to focus on the central parts of our problem, we abstract from such uncertainty.

environmental costs). We also assume that the uncertainty is resolved before households and firms act, that markets are competitive, that the government may monitor the emissions of households and firms and enforce compliance at low costs (henceforth ignored), that private costs of non-compliance are prohibitive, and that other transaction costs and income effects are negligible.

The economy consists of two sectors, Sector 1 and Sector 2, and we assume that the regulator is informed about the form (linear) but not about the location of the two sectors' MAC functions. The MAC function for sector i ($i = 1, 2$) is given by

$$(1) \quad MAC_i = \theta_i(x_i - e_i + \varepsilon_i)$$

where θ_i is a positive parameter, e_i is the emissions from sector i , and x_i denotes the sector's expected business-as-usual emission level. The cost shock to sector i (ε_i) is assumed to be distributed symmetrically around zero with the variance σ_i^2 .⁷ From these sector-wise functions we derive the aggregate *MAC* function

$$(2) \quad MAC = \theta(x - e + \varepsilon)$$

where $e = e_1 + e_2$, $x = x_1 + x_2$, and $\theta = \theta_1\theta_2/(\theta_1 + \theta_2)$. The aggregate cost shock $\varepsilon = \varepsilon_1 + \varepsilon_2$ has the mean zero and the variance $\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\sigma_{12}$, where σ_{12} denotes the covariance between the two cost shocks. The benefit of additional abatements, or equivalently the cost of additional emissions, is given by

$$(3) \quad MAB = f + ge$$

where f and g are positive parameters.

The emission level that equates the *MAB* and the *MAC* functions is given by

$$(4) \quad e^* = \frac{-f + (x + \varepsilon)\theta}{g + \theta}$$

⁷ For small uncertainties these linear functions may be seen as approximations of convex *MAC* functions (Weitzman, 1978). However, for larger uncertainties such a view may be misleading regarding both the instrumental choice and the choice of control level (Malcomson, 1978; Yohe, 1978).

At e^* , the economic value or environmental cost of an additional emission unit equals

$$(5) \quad v^* = \frac{(f+g(x+\varepsilon))\theta}{g+\theta}$$

Setting (5) equal to (1) and solving for e_i gives the *ex post* cost-effective allocation of the emission volume e^* .

$$(6) \quad e_1^*(x_1, \varepsilon_1) = \frac{\theta_1(g+\theta_2)(x_1+\varepsilon_1) - \theta_2(f+g(x_2+\varepsilon_2))}{(g+\theta)(\theta_1+\theta_2)}$$

$$(7) \quad e_2^*(x_2, \varepsilon_2) = \frac{\theta_2(g+\theta_1)(x_2+\varepsilon_2) - \theta_1(f+g(x_1+\varepsilon_1))}{(g+\theta)(\theta_1+\theta_2)}$$

Under a cap-and-trade (*CnT*) regime, the regulator sets an aggregate cap level (q) and distributes tradable emission permits to the firms and the households up to that level. From Montgomery (1972) we know that competitive permit trade equalizes *MACs* over all agents irrespectively of how the permits initially are distributed. We may thus state the social costs under an economy-wide *CnT* system as

$$(8) \quad \int_q^{x+\varepsilon} MAC(e; \varepsilon) de + \int_0^q MAB(e) de$$

where the second term is the environmental costs. After expanding the integrals and taking expectations it is straightforward to show that a cap equal to the expectation of (4) minimizes expected social cost. Under this cap (q^{opt}) expected social cost amounts to

$$(9) \quad SC_{CnT} = \frac{-f^2 + 2f\theta x + g\theta x^2}{2(g+\theta)} + \frac{\theta}{2} \sigma^2$$

The emission level under an economy-wide emission tax is obtained by setting the tax level t equal to (2) and solve for e , which gives $e(t, \varepsilon) = \frac{-t}{\theta} + x + \varepsilon$. The social cost under such a tax (*T*) regime is given by

$$(10) \quad \int_{e(t; \varepsilon)}^{x+\varepsilon} MAC(e; \varepsilon) de + \int_0^{e(t; \varepsilon)} MAB(e) de$$

It can be shown that the tax that minimizes the expectations of (10) equals the expectation of (5) and that expected social cost under this tax t^{opt} equals

$$(11) \quad SC_T = \frac{-f^2 + 2f\theta x + g\theta x^2}{2(g+\theta)} + \frac{g}{2}\sigma^2$$

The difference in expected social cost between the T and the CnT policies equals

$$(12) \quad SC_{T-CnT} = \frac{g-\theta}{2}\sigma^2$$

This expression is Weitzman's formula (in terms of our model) for the optimal choice between a tax and a cap-and-trade system, saying that the former outperforms the latter when the MAC function is steeper than the MAB function (*i.e.*, $g < \theta$) and *vice versa*, and that the instrumental choice becomes more important the larger the difference in the slope parameters is and the more uncertain we are about abatement costs.

We now turn to derive the optimal mix of an emission tax and a cap-and-trade system. Mandell (2008) notes that the regulator, by the means of such a policy mix, may induce an aggregate emission volume closer to the *ex post* efficient one than what the uniform policies are capable of. He shows that the value of this *volume effect* may outweigh the *cost-effectiveness losses* that arise because different emitters face different "prices" on emissions. Our model deviates from the one in Mandell (2008) in several ways. For instance, we treat the frontier between the two sectors as exogenously given and we assume that the firms/households within each sector are identical. Furthermore, we allow for sector-wise idiosyncratic cost shocks.

Let the emissions from Sector 1 be governed by the tax t_1 and let Sector 2 be subject to a cap-and-trade system with the cap q_2 . For this policy mix to make sense the MAC_1 function must be at least as steep as the MAC_2 function. We therefore assume that $\theta_1 \geq \theta_2$. Since we can switch the treatments of the two sectors, this assumption is not as bold as it may appear. The regulator's problem is to choose the pair (t_1, q_2) that minimizes the expectations of

$$(13) \quad \int_{e_1(t_1; \varepsilon_1)}^{x_1 + \varepsilon_1} MAC_1(e_1; \varepsilon_1) de_1 + \int_{q_2}^{x_2 + \varepsilon_2} MAC_2(e_2; \varepsilon_2) de_2 + \int_0^{e_1(t_1; \varepsilon_1) + q_2} MAB(e) de$$

where

$$(14) \quad e_1(t_1, \varepsilon_1) = \frac{-t_1}{\theta_1} + x_1 + \varepsilon_1$$

is obtained by setting t_1 equal to (1) and solving for e_1 . Since we deal with a fully mixed pollutant we can, in general, not separate the benefits of the two sectors' abatement efforts. As shown in Appendix 1 the optimal policy mix consists of the tax t_1^{opt} (equal to the expectations of (5)) and the cap q_2^{opt} (equal to the expectation of (7)). The expected social cost under the optimal policy mix (*Mix*) amounts to

$$(15) \quad SC_{Mix} = \frac{-f^2 + 2f\theta x + g\theta x^2}{2(g + \theta)} + \frac{g\sigma_1^2 + \theta_2\sigma_2^2}{2}$$

Subtracting (15) from (11) gives the difference in expected social cost between the optimal *T* policy and the optimal *Mix* policy,

$$(16) \quad SC_{T-Mix} = \frac{g-\theta}{2} \sigma_2^2 + g\sigma_1\sigma_2\rho_{12} - \frac{\theta}{2} \frac{\theta_2}{\theta_1} \sigma_2^2$$

where ρ_{12} is the correlation between the costs shocks to the two sectors. A positive value on (16) means that the policy *Mix* outperforms the *T* policy. The first two terms of (16) captures the *volume effect* of governing the emissions by a policy mix instead of an emission tax, *i.e.*, the expected gains of letting the aggregate emissions be $e_1(t_1^{opt}) + q_2^{opt}$ instead of $e(t^{opt})$. The volume effect consists here of two parts; a *Weitzman effect* (the first term) arising because we control the emissions in Sector 2 by a cap-and-trade system instead of an emission tax, and a *hedging effect* (the second term) arising because the two instruments that we mix react differently to a given shock (*i.e.*, they produce volume errors of different signs). The third term reflects the *cost-effectiveness loss* that arises because the mixed policy induces two different prices on emissions. It should be noted that the comparison with the tax regime is only relevant when $\theta \geq g$, whereby the *Weitzman effect* is non-positive.⁸ Thus, for the policy mix to outperform the tax policy the *hedging effect* must be positive, which requires a positive correlation coefficient. More precisely, the correlation must satisfy

⁸ When $\theta < g$, the tax regime is not the preferred uniform policy alternative.

$$(17) \quad \rho_{12} > .5 \frac{\theta}{g} \left(\frac{\theta-g}{\theta} + \frac{\theta_2}{\theta_1} \right) \frac{\sigma_2}{\sigma_1} \quad \text{for } \theta \geq g$$

The difference in expected social cost between the optimal *CnT* policy and the optimal *Mix* policy is obtained by subtracting (15) from (9),

$$(18) \quad SC_{CnT-Mix} = -\frac{g-\theta}{2} \sigma_1^2 + \theta \sigma_1 \sigma_2 \rho_{12} - \frac{\theta}{2} \frac{\theta_2}{\theta_1} \sigma_2^2$$

This expression is similar to (16). However, the *volume effect* now stems from that the emissions from Sector 1 are taxed instead of being covered by an economy wide cap-and-trade system. So, here the policy mix serves to increase the flexibility of the control system. For the *CnT* system to be the relevant comparison we must have $g \geq \theta$. Consequently, also here the *Weitzman effect* is non-positive. Since the *cost-effectiveness loss* is the same as in the *T-Mix* case we find that a positive *hedging effect* is a necessary condition for the policy mix to outperform the economy-wide *CnT* system. Indeed, the correlation coefficient must satisfy

$$(19) \quad \rho_{12} > .5 \left(\frac{g-\theta}{\theta} \frac{\sigma_1^2}{\sigma_1^2} + \frac{\theta_2}{\theta_1} \right) \frac{\sigma_2}{\sigma_1} \quad \text{for } g \geq \theta$$

To see why a positive correlation is a necessary condition for the policy mix to outperform the uniform policies, remember that for a given cost shock the *T* and the *CnT* policies produce *emission volume errors* of opposite signs. With a negative (positive) correlation there is a tendency for the two sectors to face cost shocks of different (same) signs. There would then be a tendency for the policy mix to induce “*sector wise volume errors*” that go in the same (opposite) direction, and hence a larger (smaller) *aggregate volume error*. It should be noted that a rather weak positive correlation may be sufficient for the *Mix* policy to outperform the uniform policy alternatives. For instance, when $g = \theta$ the regulator is indifferent between the *T* and the *CnT* policies (so (16) and (18) coincide). Given this and $\sigma_1 = \sigma_2$, the policy mix would outperform the uniform policy alternatives as long as

$$(20) \quad \rho_{12} > .5 \frac{\theta_2}{\theta_1}$$

Since we by construction have $\theta_1 \geq \theta_2$,⁹ the right-hand-side is smaller than or equal to .5. We have here shown that the optimal policy mix outperforms the uniform policies for a wider range of parameter values than those studied in Mandell (2008), who assumed that the two sectors were subjected to one and the same cost-shock.

3. Indexed Carbon Tax

We are now in the position to study a policy that index the emission tax in Sector 1 to the permit price in Sector 2. We start with a diagrammatic exposition to give the intuition for how such a policy may reduce social cost. Thereafter we derive the optimal linear indexation scheme and study its determinants and relative performance. We conclude the Section with a numerical analysis with parameter values taken from the EU climate change policy context.

Figure 1 illustrates an economy with uncertain abatement costs. Expected sector-wise *MAC* functions are illustrated by the solid lines. The departure for the analysis is the optimal policy mix t_1^{opt} and q_2^{opt} , for which expected aggregate emissions amount to $E\{e_1^*\} + q_2^{opt} = E\{e^*\}$. Assume now that the realized cost functions deviate from their expected locations as indicated by the MAC_i^{High} lines. The aggregate emissions would then equal $e_1^H + q_2^{opt}$ and thus exceed the efficient level $e^*_{ex\ post}$. Since the emissions from Sector 2 are capped, we may regard the segment of the *MAB* function that lies to the right of q_2^{opt} in Figure 1c as the marginal benefit of abatements in Sector 1 ($MAB_{Residual}$). By inserting the $MAB_{Residual}$ function in Figure 1a it becomes clear that the tax level t_1^{opt} is too low, given this cost realisation. The resulting efficiency loss equals the smaller shaded area in Figure 1a. *Ex post* we would like to have set the tax at t_1^H .¹⁰ Obviously, an indexation scheme under which the tax level depends positively on the permit price could work in this direction. Of course, a high permit price may be a false signal of high costs in Sector 1. When that is the case, this kind of indexation may give an outcome that is even worse than with no tax updating. For instance, if the costs in Sector 1 turn out to be given by MAC_1^{Low} instead of MAC_1^{High} the tax level t_1^H would generate a large efficiency loss (indicated by the larger shaded area in Figure 1a).

⁹ Remember that if this is not the case, it would be optimal to reverse the treatment of the two sectors.

¹⁰ To be sure, we would also like to adjust the cap for Sector 2. However, by assumption this is not a valid policy option here.

{Figure 1 about here.}

We now turn to a formal analysis of linear tax indexation schemes, *i.e.*, updating rules of the form $t_1 = \alpha + \beta[E\{p_2\} - p_2]$. Given a competitive emission permit trading in Sector 2, we have $p_2 = MAC_2(q_2; \varepsilon_2) = \theta_2(x_2 + \varepsilon_2 - q_2)$. Using this relationship to eliminate p_2 and $E\{p_2\}$ we get $t_1 = \alpha - \beta\theta_2\varepsilon_2$. Inserting this expression in (14) gives

$$(21) \quad e_1(\alpha, \beta, \varepsilon_1, \varepsilon_2) = \frac{-(\alpha - \beta\theta_2\varepsilon_2)}{\theta_1} + x_1 + \varepsilon_1$$

This is the emission level in Sector 1 under this updating. We can now state the regulator's problem as one of choosing the triplet $(\alpha, \beta$ and $q_2)$ that minimizes the expectations of

$$(22) \quad \int_{e_1(\alpha, \beta, \varepsilon_1, \varepsilon_2)}^{x_1 + \varepsilon_1} MAC_1(e_1; \varepsilon_1) de_1 + \int_{q_2}^{x_2 + \varepsilon_2} MAC_2(e_2; \varepsilon_2) de_2 + \int_0^{e_1(\alpha, \beta, \varepsilon_1, \varepsilon_2) + q_2} MAB(e) de$$

From the *FOCs* of this problem we have.¹¹

$$(23) \quad \alpha^{opt} = \frac{\theta}{g + \theta}(f + gx)$$

$$(24) \quad \beta^{opt} = \frac{-g}{g + \theta_1} \frac{\theta_1 \sigma_1}{\theta_2 \sigma_2} \rho_{12}$$

$$(25) \quad q_2^{Ind} = \frac{\theta_2(g + \theta_1)x_2 - \theta_1(f + gx_1)}{(g + \theta)(\theta_1 + \theta_2)} = E\{e_2^*\}$$

Inserting (23) and (24) in (21) gives the optimal indexed tax formula

$$(26) \quad t_1^{Ind} = \frac{\theta}{g + \theta}(f + gx) + \frac{g}{g + \theta_1} \theta_1 \frac{\sigma_1}{\sigma_2} \rho_{12} \varepsilon_2$$

The second term of (26) is the adjustment of the tax level due to the index rule. The direction and the size of this adjustment depend on the correlation coefficient. A negative (positive)

¹¹ For derivations, see Appendix 2.

correlation implies that the tax level is adjusted in the opposite (same) direction as the permit price deviates from its expected level. It is easy to see that $E\{t_1^{Ind}\} = t_1^{opt} = E\{v^*\}$. It may also be noted that the steeper the MAB and MAC_I functions are, the larger is the tax adjustment for a given cost shock. The same goes for the uncertainty surrounding the location of the MAC_I curve.

Expected social cost under the optimal index policy amounts to

$$(27) \quad SC_{Ind} = \frac{-f^2 + 2f\theta x + g\theta x^2}{2(g + \theta)} + \frac{g\sigma_1^2 + \theta_2\sigma_2^2}{2} - \frac{g^2}{2(g + \theta_1)}\rho_{12}^2\sigma_1^2$$

The difference in expected social costs between the policy mix and the indexation policy is obtained by subtracting (27) from (15).

$$(28) \quad SC_{Mix-Ind} = \frac{g^2}{2(g + \theta_1)}\rho_{12}^2\sigma_1^2$$

Eq. (28) is positive as long as there is some correlation between the cost shocks to the two sectors. In other words, the index policy outperforms the policy mix as long as the permit price conveys any information about the likely location of the MAC_I function. This result should come as no surprise. After all, we have assumed that the regulator knows the correlation and is equipped with an instrument to utilize this information (an adjustable emission tax). Eq. (28) also tells us that the expected gains of indexing the tax level in Sector 1 to the permit price is larger the stronger is the correlation coefficient, the steeper is the MAB function (g), and the more uncertain we are about the location of the MAC_I function. Furthermore, the steeper the MAC_I function the less can the regulator influence the emission volume by adjusting the tax level, and the smaller the gains become. Also these results are in line with our intuition.

We now compare the index policy with the uniform policy alternatives. Subtracting (27) from (9) and (11), respectively, gives the difference in expected social costs between the CnT and the Ind policies and the T and Ind policies.

$$(29) \quad SC_{CnT-Ind} = -\frac{g-\theta}{2}\sigma_1^2 + \theta\sigma_1\sigma_2\rho_{12} - \frac{\theta}{2}\frac{\theta_2}{\theta_1}\sigma_2^2 + \frac{g^2}{2(g+\theta_1)}\rho_{12}^2\sigma_1^2 \quad \text{for } g \geq \theta$$

$$(30) \quad SC_{T-Ind} = \frac{g-\theta}{2} \sigma_2^2 + g \sigma_1 \sigma_2 \rho_{12} - \frac{\theta}{2} \frac{\theta_2}{\theta_1} \sigma_2^2 + \frac{g^2}{2(g+\theta_1)} \rho_{12}^2 \sigma_1^2 \quad \text{for } g \leq \theta$$

The first three terms of (29) and (30) are familiar from eqs. (16) and (18). These terms reflect the net-gain (the sum of the *Weitzman effect*, the *hedging effect* and the *cost-effectiveness loss*) of going from the relevant uniform policy to the optimal policy mix. The fourth term captures the *gain of indexing* the tax level depend on the permit price. This effect equals eq. (28) and is nonnegative. Hence, the optimal index policy outperforms the uniform policies over a wider range of parameter values than the optimal policy mix does. Still, for a large parameter set the index policy is outperformed by the uniform policy alternatives. This is easily seen by assuming zero correlation. Then (29) and (30) become negative.

More generally, for SC_{T-Ind} to be positive we must have (see Appendix 3 for derivation)

$$(31) \quad \rho_{12} \geq \frac{g+\theta_1}{g} \left(\sqrt{\frac{\theta_1+\theta_2}{g+\theta_1}} - 1 \right) \frac{\sigma_2}{\sigma_1} \quad \text{for } g \leq \theta$$

Since $g \leq \theta < \theta_2 \leq \theta_1$, the square root exceeds one and the right-hand side is positive. In other words, a strictly positive correlation is required for the index policy to outperform the tax alternative.

A positive $SC_{CnT-Ind}$ requires (see Appendix 3 for derivation)

$$(32) \quad \rho_{12} \geq \frac{g+\theta_1}{g} \left(\sqrt{\frac{g-\theta}{g+\theta_1} \frac{\sigma_1^2}{\sigma_2^2} + \frac{\theta^2}{g^2} + \frac{\theta}{g+\theta_1} \frac{\theta_2}{\theta_1} - \frac{\theta}{g}} \right) \frac{\sigma_2}{\sigma_1} \quad \text{for } g \geq \theta$$

Since the square root exceeds θ/g , a positive correlation is required also for the index policy to outperform the cap-and-trade approach.

Even though the expressions (31) and (32) are a bit messy the intuition behind them are basically the same as the one behind the comparisons between the *Mix* policy and the uniform policy alternatives. Under the assumption that $\theta = g$, expected social costs of the *T* regime equal those under *CnT* policy (*i.e.*, (29) and (30) coincide). Then, the index policy outperforms the uniform policy alternatives if

$$(33) \quad \rho_{12} \geq \frac{\theta+\theta_1}{\theta} \left(\sqrt{\frac{(\theta_1+\theta_2)^2}{(\theta_1+\theta_2)^2 - \theta_2^2}} - 1 \right) \frac{\sigma_2}{\sigma_1}$$

Further assuming that $\sigma_1 = \sigma_2$ and $\theta_1 = \theta_2$ we find that the correlation must exceed .46 for the index policy to outperform the uniform policy alternatives. Thus, at least in this case there is a substantial window of opportunity for the regulator to reduce expected social cost by the means of the index policy.

The expected net-gains of the index policy depend on parameters that may vary over environmental problems. Below we present a numerical analysis based on parameter values taken from the European climate-policy context. These values are presented in Table 1 below. The EU is of particular interest here since it for some time has had a policy mix of the kind in focus here and, as mentioned in the Introduction, it has been suggested that the future (minimum) carbon tax level in the EU should be indexed to the permit price in EU ETS.

Table 1. Parameter Values from the EU Climate Policy Context

Parameter	θ_1 €/ton CO ₂	θ_2 €/ton CO ₂	θ €/ton CO ₂	σ_1^2 Mton CO ₂	σ_2^2 Mton CO ₂
	.38	.14	~.102	526	526

The figures in Table 1 come from different sources. We obtain the slope parameters for the marginal abatement costs functions by the means of linear approximations of the cost curves for the transport sector and the EU ETS presented in Blom *et al.* (2007).¹² The values for θ_1 and θ_2 indicate how much the abatement costs increase for every additional million ton carbon dioxide (CO₂) abated in Sector 1 and Sector 2, respectively. From these we calculate the slope parameter for the whole EU (θ). Data on which we directly can assess σ_1^2 and σ_2^2 – the uncertainty regarding the business-as-usual emission levels of the two sectors – are not available. Instead we use EU ETS price data to estimate $\text{Var}(p_2)$, from which we then infer σ_2^2 .¹³ Since the consumption of fossil fuels in Sector 1 varies, we cannot use data on oil prices

¹² We approximate over the emission interval $e_i(\text{€}50)$ to $e_i(\text{€}0)$ and assume that the *MAC* for the transport sector resembles the *MAC* for the whole non-EU ETS sector.

¹³ From EU ETS spot prices for the period 2009-2012 (available at EEX (2012)) we estimate $\text{Var}(p_2)$ to 10.31. On a competitive permit market, $p_2 = \text{MAC}_2(q_2; \varepsilon_2)$. So $\text{Var}(p_2) = \theta_2^2 \text{Var}(\varepsilon_2) = \theta_2^2 \sigma_2^2$. Inserting $\theta_2 = .14$ in $10.31 = \theta_2^2 \sigma_2^2$ and solving for σ_2^2 gives the value stated in the Table.

or the gasoline price to estimate σ_1^2 in a proper manner. We therefore assume equal variances for the two sectors. This is of minor importance for the results since σ_1^2 only enters as a scale variable. More important for the results is the slope of the *MAB* function (g), a parameter surrounded by large uncertainty. We therefore consider a wide interval of values for it. It may be argued that there is a positive correlation between the cost shocks to the EU ETS and the taxed sector in EU. However, below we study the relative performance of the index policy over all possible correlation coefficients. By doing so and by varying the relative steepness of the *MAB* function we are able to draw some fairly general conclusions.

Consider Figure 2. The shaded area therein illustrates the combinations of ρ_{12} and g for which the index policy outperforms the tax policy. Since θ is held fixed, we allow the relative steepness of the *MAB* function (g/θ) to vary from around zero to 10. As mentioned above, the comparison between a uniform tax policy and the index policy is only relevant if $g/\theta \leq 1$.¹⁴ We thus focus on the area below the horizontal line in the Figure. As indicated by the vertical line the correlation must exceed .2 for the index policy to outperform the emission tax. The correlation threshold value increases fast as we reduce the (relative) slope of the *MAB* function. This is explained by the fact that as we reduce g both the value of going from a uniform tax policy to the policy mix (eq. 16) and the value of taking the additional step of indexing the tax level to the permit price (eq. 28) decrease.¹⁵ We also see that for sufficiently flat *MAB* functions ($g/\theta \leq .5$), the index policy cannot outperform the uniform emission tax no matter how strong the correlation between the cost shocks to the two sectors is.

Figure 3 shows the set of ρ_{12} and g for which the index policy outperforms an economy-wide cap-and-trade system, a comparison that only is relevant when $g \geq \theta$. When g/θ equals one the correlation coefficient must exceed .2, just as in Figure 2. This makes sense since the expected social cost of the two uniform policy alternatives then are the same. In contrast to Figure 2, here the steeper the *MAB* function is the higher must the correlation be for the indexation policy to outperform the cap-and-trade policy. It may be noted that as we are increasing g (thus increasing the relative steepness of the *MAB* function, g/θ , from unity), the threshold value of the correlation coefficient grows initially in a slower pace than in the *Ind* vs. *T* case. This is explained by the fact that the value of going from an economy-wide cap-

¹⁴ That the optimal index policy performs better than a uniform emission tax when the tax is a sub-optimal instrumental choice is of minor interest here.

¹⁵ $\frac{\partial SC_{T-Mix}}{\partial g} > 0$ for $\rho_{12} > 0$ and $\frac{\partial SC_{Mix-Ind}}{\partial g} > 0$.

and-trade system to the policy mix (eq. 18) falls in g while the gain of the additional step of indexing the tax level (eq. 28) increases in the same parameter.¹⁶ When the MAB function is more than eight times as steep as the MAC function it is impossible for the index policy to outperform the cap-and-trade system.

Figure 2. Tax vs. the Index Policy ($SC_T - SC_{Ind} \geq 0$ indicated by the blue area)

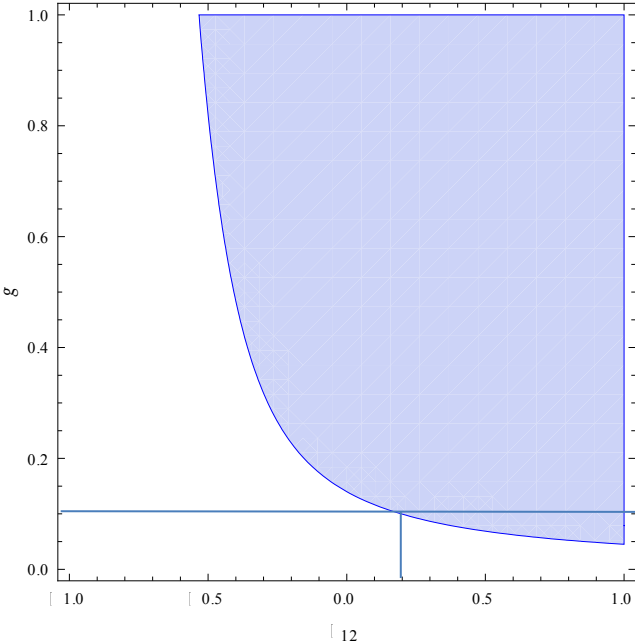
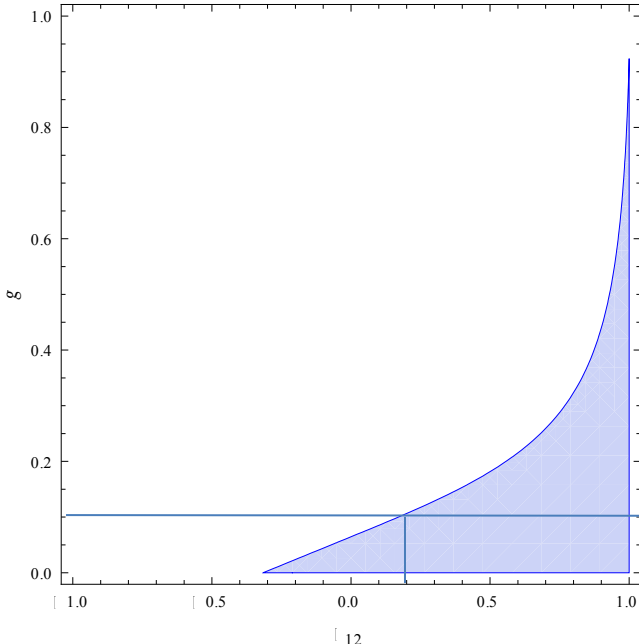


Figure 3. Cap-and-trade vs. the Index Policy ($SC_{CnT} - SC_{Ind} \geq 0$ indicated by the blue area)



¹⁶ $\frac{\partial SC_{CnT-Mix}}{\partial g} < 0$ and $\frac{\partial SC_{Mix-Ind}}{\partial g} > 0$.

The graphical representation above indicates that there exist a substantial set of parameters values for which the index policy is the most efficient control instrument.

Below we present calculations of the expected relative net-gains of using the index policy instead of (i) the policy mix, (ii) a uniform emission tax, and (iii) an economy-wide cap-and-trade system. In order to do so, we need to assume values also for the intercept of the *MAB* function (f) and the expected business-as-usual emission level (x). Regarding the latter we use as a proxy the EU's emission level in 2005. According to Capros *et al.* (2008) these amounted to 4,267 million tons CO₂, of which the EU ETS sectors accounted for 2,193 million tons of CO₂. When it comes to f , we don't have much information.¹⁷ We therefore simply assume the value €10 per ton CO₂.

Figure 4 shows the expected relative *gain of indexing* the tax in Sector 1 to the permit price in Sector 2. More precisely we graph $(SC_{Mix}-SC_{Ind})/SC_{Mix}$ over the correlation coefficient for three different g values. In line with the analytical results above, the index policy strictly outperforms the policy mix as long as the cost shocks to the two sectors are correlated. We see that the expected relative gains are rather small, not exceeding .003 percent. To a large extent this is explained by rather small absolute expected net-gains; for $g/\theta = 1$ and perfect correlation they amount to €5.8 million per year.¹⁸ However, given a good perception of the correlation between the cost-shocks to the two sectors the regulator cannot do worse in expectation.

¹⁷ Most estimates of the social cost of carbon are conducted over the range of substantial emission volumes. Also in these regions there is a large uncertainty surrounding the value, see, *e.g.* Tol (2008).

¹⁸ It may be argued that we have underestimated the degree of uncertainty. By inferring σ_2^2 from the estimated variance of the EU ETS price during 2008-12 and by assuming $\sigma_1^2 = \sigma_2^2$, we get $\sigma^2 = 1,052 + 2\sigma_{12} = 1,052 + 2\sigma_2^2\rho_{12} = 1,052(1 + \rho_{12})$. Thus, σ^2 cannot exceed 2,104, implying a (maximal) standard deviation equal to 46 Mton CO₂ (*i.e.* around one percent of the assumed expected business-as-usual emission level).

Figure 4. Expected relative gain of indexing the tax to the permit price

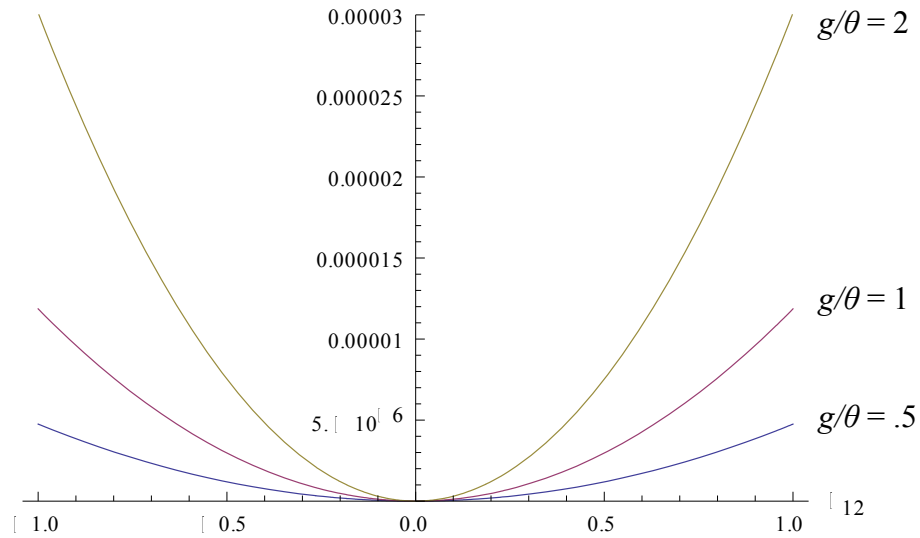


Figure 5 graphs the expected relative net-gains of going from a tax regime to the index policy – $(SC_T - SC_{Ind})/SC_T$ – over the correlation coefficient for two relevant values on the relative steepness of the *MAB* function, *i.e.*, $g/\theta = \{.5 \text{ and } 1\}$. We see that the expected gains of going from an economy-wide emission tax to the index policy are almost linear in the correlation coefficient and that they are most pronounced when the *MAC* and *MAB* functions are equally steep. For flatter *MAB* functions, expected net-gains rapidly become insignificant or negative also for very high correlation coefficients.

With a relative steepness of the *MAB* function equal to one, there is no *Weitzman effect* of going from the *T* regime to the index policy, *i.e.*, the first term of (30) equals zero. And, when there is no correlation between the cost shocks, there is no *hedging effect* of the policy mix or any *value of indexing* the tax level to the permit price, *i.e.*, the second and the fourth terms of (30) equal zero. Thus, the vertical distance from *origo* to the “ $g/\theta = 1$ graph” can be interpreted as the expected *cost-effectiveness loss* of going from a uniform emission tax to the mixed or indexed policy. The additional distance to the “ $g/\theta = .5$ graph” amounts to the *Weitzman effect* for this relative steepness of the *MAB* function. When we move to the right on the *x*-axis the *hedging effect* (the second term of (30)) becomes positive and there are *gains of indexing* the tax level to the permit price (fourth term of (30)). The circumstance that the “ $g/\theta = 1$ graph” increase faster in the correlation coefficient than the “ $g/\theta = .5$ graph” is explained by that *g* is smaller in the latter case, which *ceteris paribus* gives a smaller hedging effect and gains of indexing.

Figure 5. Expected relative gains of using the index policy instead of a uniform tax policy

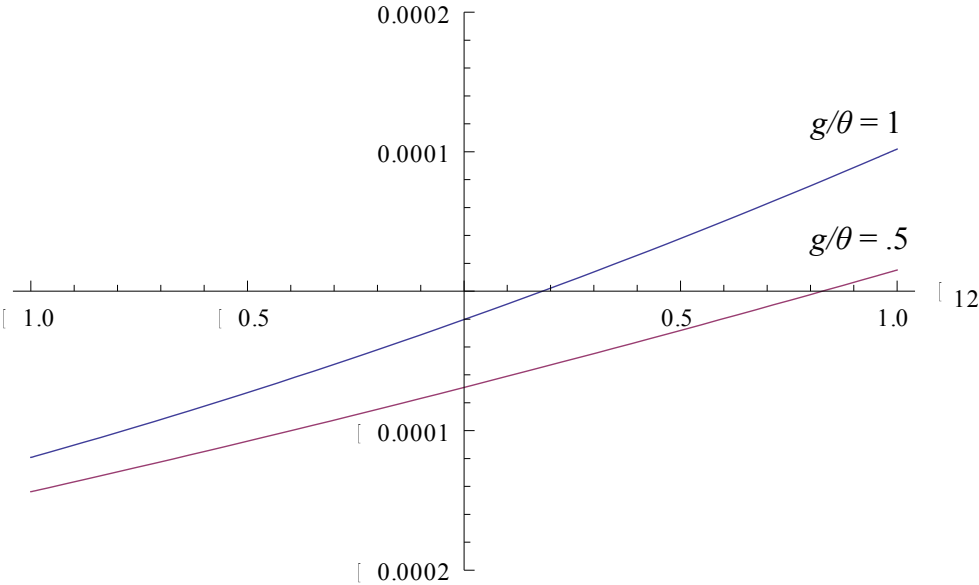
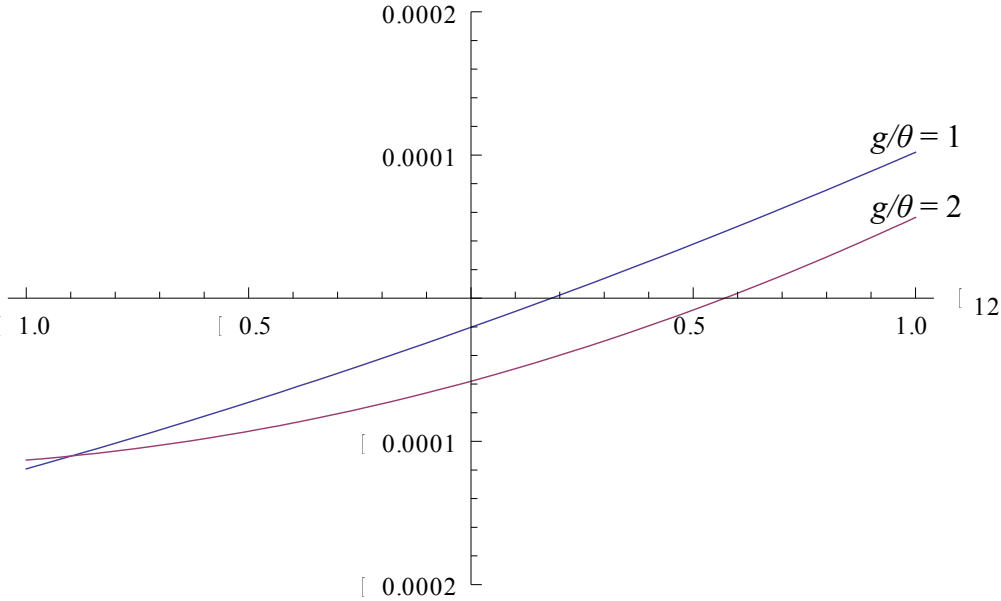


Figure 6 graphs the relative net-gains of substituting an economy-wide cap-and-trade system for the index policy, *i.e.*, $(SC_{CnT} - SC_{Ind}) / SC_{CnT}$. A picture similar to the one in Figure 5 emerges. Also here we see that the expected relative net-gains become smaller when the relative steepness of the *MAB* function deviates from unity, albeit at a somewhat slower pace than in the *Ind-T* comparison above. The *expected cost-effectiveness* loss (the third term of eq. (29)) is the same as above. And, as we move to the right of the x-axis, the *hedging effect* and the *value of indexing* increase. Notably the required correlation for the “ $g/\theta = 2$ case” is weaker than in the tax-indexation case above. This is explained by the circumstance that g is here larger than in the tax comparison case, which gives a higher value of indexing.

Figure 6. Expected relative net-gains of the index policy instead of a cap-and-trade system



The index policy may indeed outperform the uniform policy alternatives. This is in particular so for environmental problems where the *MAB* and *MAC* functions have about the same slope. Then, it is sufficient with a fairly low positive correlation between the cost-shocks to the two sectors. However, when the relative steepness of the *MAB* function substantially deviates from unity, the correlation must be strong, perhaps unrealistically so. The policy mix and the index policy studied here therefore do not seem to be strong candidates for controlling so-called stock externalities, environmental problems for which the *MAB* function is expected to have a negligible slope.

4. Concluding Remarks

We have here studied the properties of an environmental policy that (i) tax one group of emitters and let the other emitters be covered by a cap-and-trade system and (ii) index the tax level to the permit price. Effectively, such a policy updates the tax level with respect to the cost-shock hitting the capped sector. This could be of value given that the regulator is informed about the correlation between the cost-shocks hitting the two sectors. We find that the optimal linear index policy outperforms the optimal policy mix studied in Mandell (2008) as long as there is any correlation. Thus, in economies that already use a mix of an emission tax and cap-and-trade system the regulator may further reduce expected social costs by

indexing the tax level to the permit price. The index policy may also outperform the uniform policy alternatives (an economy-wide tax and an economy-wide cap-and-trade system). The conditions for this are more restrictive, though. First, the correlation between the cost shocks to the two sectors must be positive. Second, when the relative steepness of the marginal abatement benefit function substantially deviates from unity the correlation coefficient must be very strong. Thus, the indexation policy studied here does not seem well-suited for controlling so-called stock externalities, which generally are expected to have flat *MAB* functions.

The climate policy in the EU has been designed as a policy mix of the kind studied here. Setting aside political problems, the analysis above indicate that it would be possible to further reduce expected social cost by taking the additional step of indexing the member states' carbon tax levels to the permit price. However, given parameters values taken from the European climate change context the expected relative gains of doing so are rather small. The optimal uniform policy alternative (an emission tax) outperforms both the policy mix and the index policy for realistic correlation coefficients. The circumstances that the regulator may have only a vague perception of the correlation and an opportunity to adjust the level of an economy-wide carbon tax over time further strengthen the argument for a uniform carbon tax.

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Appendix 1. Derivation of the optimal policy mix (t_1^{opt} , q_2^{opt})

Insert MAC_1 , MAC_2 , (3), and (14) in (13). Expand and take expectations. After some manipulation we have

(A1.1)

$$\frac{t_1^2}{2\theta_1} + \frac{\theta_2}{2}(q_2^2 - 2q_2x_2 + x_2^2 + \sigma_2^2) + \frac{gt_1^2 - 2t_1(f + gq_2 + gx_1)\theta_1 + \theta_1^2(gq_2^2 + 2fx_1 + gx_1^2 + 2q_2(f + gx_1) + g\sigma_1^2)}{2\theta_1^2}$$

This is the expected social cost of the policy mix t_1 and q_2 . Let $h(t_1, q_2)$ denote (A1.1). The first order conditions for the problem of minimizing $h(t_1, q_2)$ with respect to t_1 and q_2 are:

$$(A1.2) \quad \frac{\partial h(t_1, q_2)}{\partial t_1} = -\frac{f + g(q_2 - t_1 + x_1) - t_1\theta_1}{\theta_1^2} = 0$$

$$(A1.3) \quad \frac{\partial h(t_1, q_2)}{\partial q_2} = f + gq_2 + gx_1 - \frac{gt_1}{\theta_1} + (q_2 - x_2)\theta_2 = 0$$

Solving for t_1 and q_2 , gives

$$(A1.4) \quad t_1^{opt} = \frac{f + gx_1}{g + \theta} \theta$$

$$(A1.5) \quad q_2^{opt} = \frac{(g\theta_2 + \theta_1\theta_2)x_2 - (f + gx_1)\theta_1}{(g + \theta)(\theta_1 + \theta_2)}$$

which equal the expectations of (5) and (7), respectively. Inserting (A1.4) and (A1.5) in (A1.1), yields after some manipulation (15). The second partial derivatives of (A1.1) amounts to

$$(A1.6) \quad \begin{bmatrix} \frac{\partial^2 h(t_1, q_2)}{\partial^2 t_1} & \frac{\partial^2 h(t_1, q_2)}{\partial t_1 \partial q_2} \\ \frac{\partial^2 h(t_1, q_2)}{\partial q_2 \partial t_1} & \frac{\partial^2 h(t_1, q_2)}{\partial^2 q_2} \end{bmatrix} = \begin{bmatrix} \frac{g}{\theta_1^2} + \frac{1}{\theta_1} & -\frac{g}{\theta_1} \\ -\frac{g}{\theta_1} & g + \theta_2 \end{bmatrix}$$

Since the leading principal minors are positive, the SOCs of our minimization problem are satisfied.

Appendix 2. Derivation of the optimal linear index policy $(\alpha^{opt}, \beta^{opt}, q_2^{opt})$

Insert MAC_1 , MAC_2 , (3), and (21) in (22). Expand and take expectations. Since $E\{\varepsilon_1\} = E\{\varepsilon_2\} = 0$, we have that $\sigma_{12} = E\{\varepsilon_1\varepsilon_2\}$. Using this we get after some manipulation

$$(A2.1) \quad \frac{\alpha^2}{2\theta_1} + \frac{\beta^2\theta_2^2}{2\theta_1}\sigma_2^2 + \frac{\theta_2}{2}(q_2^2 - 2q_2x_2 + x_2^2 + \sigma_2^2) + \frac{\theta_1^2((q_2 + x_1)(2f + g(q_2 + x_1)) + g\sigma_1^2) + g(\alpha^2 + \beta^2\theta_2^2\sigma_2^2) - 2\theta_1(\alpha(f + g(q_2 + x_1)) - g\beta\theta_2\sigma_{12})}{2\theta_1^2}$$

This is the expected social cost under the index policy. Let $h(\alpha, \beta, q_2)$ denote (A2.1). The first order conditions for the problem of minimizing (A2.1) with respect to α , β and q_2 , are:

$$(A2.2) \quad \frac{\partial h(\alpha, \beta, q_2)}{\partial \alpha} = \frac{\alpha}{\theta_1} + \frac{g\alpha - (f + g(q_2 + x_1))\theta_1}{\theta_1^2} = 0$$

$$(A2.3) \quad \frac{\partial h(\alpha, \beta, q_2)}{\partial \beta} = \frac{\beta\theta_2^2\sigma_2^2}{\theta_1} + \frac{g\theta_2(\beta\theta_2\sigma_2^2 + \theta_1\sigma_{12})}{\theta_1^2} = 0$$

$$(A2.4) \quad \frac{\partial h(\alpha, \beta, q_2)}{\partial q_2} = \frac{-g\alpha + (f + g(q_2 + x_1))\theta_1}{\theta_1} + (q_2 - x_2)\theta_2 = 0$$

Solving for α^{opt} , β^{opt} and q_2^{Index} gives (23), (24) and (25).

The second partials of (A2.1) are

$$(A2.5) \quad \begin{bmatrix} \frac{\partial^2 h(\alpha, \beta, q_2)}{\partial^2 \alpha} & \frac{\partial^2 h(\alpha, \beta, q_2)}{\partial \alpha \partial \beta} & \frac{\partial^2 h(\alpha, \beta, q_2)}{\partial \alpha \partial q_2} \\ \frac{\partial^2 h(\alpha, \beta, q_2)}{\partial \beta \partial \alpha} & \frac{\partial^2 h(\alpha, \beta, q_2)}{\partial^2 \beta} & \frac{\partial^2 h(\alpha, \beta, q_2)}{\partial \beta \partial q_2} \\ \frac{\partial^2 h(\alpha, \beta, q_2)}{\partial q_2 \partial \alpha} & \frac{\partial^2 h(\alpha, \beta, q_2)}{\partial q_2 \partial \beta} & \frac{\partial^2 h(\alpha, \beta, q_2)}{\partial^2 q_2} \end{bmatrix} = \begin{bmatrix} \frac{g}{\theta_1^2} + \frac{1}{\theta_1} & 0 & -\frac{g}{\theta_1} \\ 0 & \frac{g\theta_2^2\sigma_2^2}{\theta_1^2} + \frac{\theta_2^2\sigma_2^2}{\theta_1} & 0 \\ -\frac{g}{\theta_1} & 0 & g + \theta_2 \end{bmatrix}$$

Since all the leading minors are positive, the *SOCs* of our minimization problem are fulfilled.

Appendix 3. Derivation of (31) and (32).

Set (29) equal to zero and solve for ρ_{12} . Then, after using the circumstance that $\theta = \theta_1\theta_2/(\theta_1+\theta_2)$ and some manipulation we get

$$\text{T: (A3.1a)} \quad \rho_{12} = -\frac{g+\theta_1}{g} \left(1 + \sqrt{\frac{\theta_1+\theta_2}{g+\theta_1}} \right) / \frac{\sigma_1}{\sigma_2}$$

$$\text{T: (A3.1b)} \quad \rho_{12} = \frac{g+\theta_1}{g} \left(-1 + \sqrt{\frac{\theta_1+\theta_2}{g+\theta_1}} \right) / \frac{\sigma_1}{\sigma_2}$$

Since $g \leq \theta < \theta_2 \leq \theta_1$, the expression under the square root exceeds one. This means that unless $\frac{\sigma_1}{\sigma_2} < 2 \frac{g+\theta_1}{g}$ the RHS of (A3.1a) must be smaller than *minus* one. We therefore dismiss this root. The remaining root is (31).

Repeating the procedure above for eq. (31) gives

$$\text{CnT: (A3.2a)} \quad \rho_{12} = -\frac{g+\theta_1}{g} \left(\frac{\theta}{g} + \sqrt{\frac{g-\theta}{g+\theta_1} \frac{\sigma_1^2}{\sigma_2^2} + \frac{\theta^2}{g^2} + \frac{\theta}{g+\theta_1} \frac{\theta_2}{\theta_1}} \right) / \frac{\sigma_1}{\sigma_2}$$

$$\text{CnT: (A3.2b)} \quad \rho_{12} = -\frac{g+\theta_1}{g} \left(-\frac{\theta}{g} + \sqrt{\frac{g-\theta}{g+\theta_1} \frac{\sigma_1^2}{\sigma_2^2} + \frac{\theta^2}{g^2} + \frac{\theta}{g+\theta_1} \frac{\theta_2}{\theta_1}} \right) / \frac{\sigma_1}{\sigma_2}$$

Here we have $g \geq \theta < \theta_2 \leq \theta_1$. Consider the case when $g = \theta$. Then, the square root exceeds one. Thus, at least for $\frac{\sigma_1}{\sigma_2} < 2 \frac{g+\theta_1}{g}$ the RHS of (A3.2a) is smaller than *minus* one. We therefore dismiss this root. The remaining root is (32).

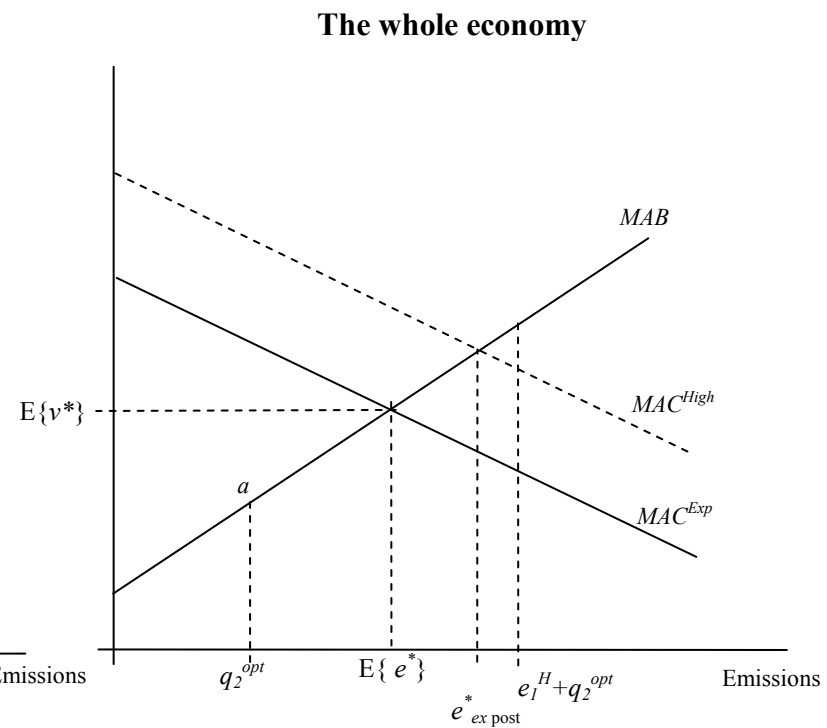
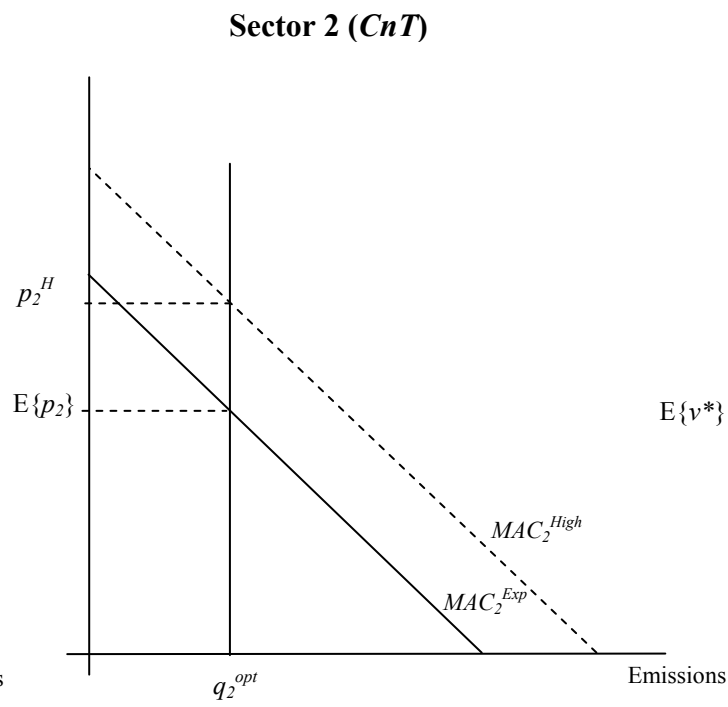
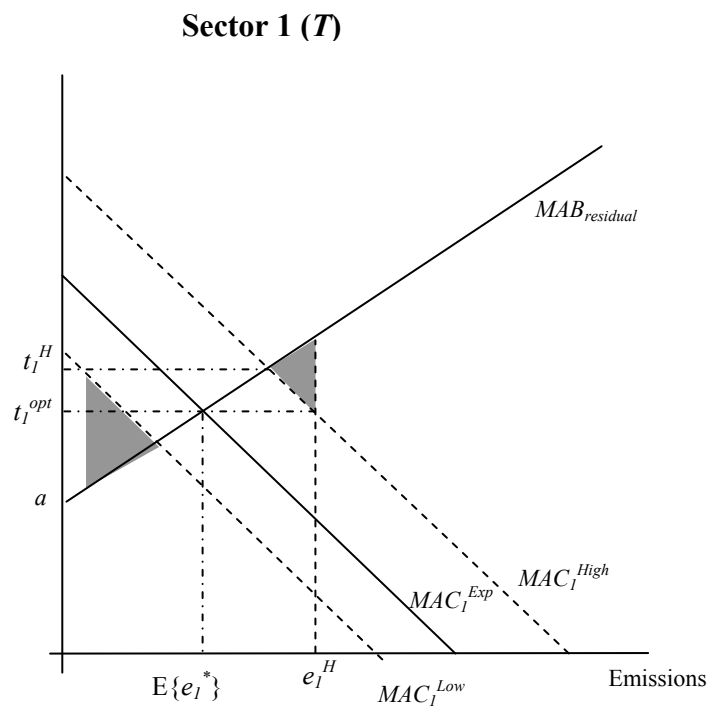


Figure 1. The potential value of adjusting the tax level in Sector 1.