Marginal Railway Renewal Costs:

A Survival Data Approach

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Abstract

In this paper, renewal costs for railway tracks are investigated using survival analysis. The purpose is to derive the effect from increased traffic volumes on rail renewal cycle lengths and to calculate associated marginal costs. A flow sample of censored data containing almost 1 500 observations on the Swedish main railway network is used. We specify Weibull accelerated failure time regression models, and estimate deterioration elasticities for total tonnage as well as for passenger and freight tonnages separately. Marginal costs are calculated as a change in present values of renewal costs from premature renewal following increased traffic volumes.

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1.0 Introduction

Pricing infrastructure wear and tear is of great importance from an efficiency standpoint. Over the last decade, research on the subject has gradually increased for all modes of transport, both in Sweden and internationally (Nash and Sansom, 2001; Nash, 2003; Thomas *et al*., 2003; Bruzelius, 2004; Nash and Matthews, 2005). Sweden has a long tradition of marginal cost pricing in the transport sector, but to date, railway infrastructure wear and tear fees have excluded costs for rail renewal. This issue has drawn some attention recently regarding the lack of empirical evidence on the size of a pricing relevant rail renewal cost component (Nash, 2005).

The fee for railway wear and tear in Sweden is based on econometric analyses of infrastructure operation and maintenance costs by Johansson and Nilsson (2004) and Andersson (2006). Johansson and Nilsson (2004) use cost data from the mid 1990´s, but detailed information on renewals was not available at the time of their analysis. Andersson (2006, 2007) extends their analysis with data from 1999 to 2002 by including renewal costs in analyses using initially pooled ordinary least squares (POLS) and later fixed effects (FE) models. He finds a higher cost elasticity with respect to output than Johansson and Nilsson (2004) and increased marginal costs in the POLS approach, but a lower cost elasticity in the FE approach. The POLS approach was rejected and furthermore, the cost function approach to identify the marginal cost of renewals is questioned. As rail renewals have long life cycles (are rare events), the lack of comprehensive time-series data questions the adequacy of applying traditional regression analysis to the renewal problem. In contrast to previous applications of econometric techniques, we suggest a different approach in this paper using an analytical expression for the marginal rail renewal cost and survival analysis as input to the marginal cost calculation.

Almost 1 500 railway track segment observations are used to analyse rail life in relation to freight and passenger traffic in Sweden. Weibull survival functions are estimated using rail life as dependent variable while traffic and other infrastructure variables are used as covariates.

The main findings are that the estimated models give a good fit of the data. We find strong positive duration dependence, with a more than proportional increase in renewal risk over time with respect to accumulated traffic. This supports the choice of the Weibull model for this data. The elasticity of rail age with respect to traffic is higher for freight trains, than for passenger trains. The marginal costs for freight and passenger trains are estimated to approximately SEK 0.002-3 per gross ton kilometre[1].

The outline of the paper is as follows. In section 2 we describe the modelling approach followed by a data review in section 3. The model results are presented in section 4. Marginal costs are calculated in section 5 while section 6 concludes.

2.0 The modelling approach

The modelling is separated into two separate stages. We initially draw the framework for rail life modelling followed by the approach to marginal cost derivation.

2.1 Modelling rail life

Survival data is used in a number of research disciplines. In medicine, it can be the case of time elapsed between a treatment and a specific health state. In labour economics, it can be

[1] The exchange rate from Swedish Kronor (SEK) to Euro (EUR) is SEK 9.32/EUR and from Swedish Kronor to US Dollar (USD) is SEK 6.89/USD (August 20, 2007).

the time spell of a person being unemployed or in engineering, the time until a component fails to perform its intended function.

In this paper, the expected life of railway track segments is analysed using parametric survival models. The general theory and concepts of survival analysis and model estimation is well developed and can be found in Kiefer (1988), Lancaster (1992) or Klein and Moeschberger (2002). We will follow the terminology of Klein and Moeschberger (2002) in the presentation of the underlying theory of the survival analysis.

Let X be a nonnegative random variable, representing the time in years between two railway track segment renewals that is rail life. There are a few different ways of characterising the distribution of X and if we know any of these, the others can be identified. First, the distribution of X can be represented by a survival function. The survival function S states the probability P of an individual track segment surviving beyond time x.

$$S(x) = P(X > x) \tag{1}$$

The survival function is the complement to the cumulative distribution function F, $S(x) = 1 - F(x)$, where $F(x) = P(X \leq x)$. The probability density function f gives the unconditional probability of a track segment being renewed in time x, $f(x) = -\frac{dS(x)}{dx}$.

Second, the probability that a track segment of age x will be renewed instantaneously after x ($x + \Delta x$) is represented by the hazard rate h. The hazard function for a continuous variable is defined by

$$h(x) = \lim_{\Delta x \to 0} \frac{P[x \leq X < x + \Delta x \mid X \geq x]}{\Delta x} = \frac{f(x)}{S(x)} = \frac{-d \ln[S(x)]}{dx}. \tag{2}$$

The cumulative (or integrated) hazard function *H* is defined as

$$H(x) = \int_0^x h(u)du = -\ln[S(x)]. \tag{3}$$

The probability density function $f(x)$ can also be expressed using the hazard function and the cumulative hazard as in (4).

$$f(x) = h(x)\exp[-H(x)] \tag{4}$$

The Weibull model is used in the analyses, which is a popular parametric model for engineering survival data. The model has a survival function $S(x) = \exp[-\lambda x^{\alpha}]$ for $x > 0$. $\lambda > 0$ and $\alpha > 0$ are known as the scale and shape parameters respectively. The hazard rate has the form of $h(x) = \lambda\alpha x^{\alpha-1}$ and the cumulative hazard $H(x) = \lambda x^{\alpha}$. The probability density function and cumulative distribution function are expressed as $f(x) = \lambda\alpha x^{\alpha-1}\exp[-\lambda x^{\alpha}]$ and $F(x) = 1-\exp[-\lambda x^{\alpha}]$ respectively. Finally, μ is the expected value of the renewal interval, $E(X) = \mu = \frac{\Gamma(1+1/\alpha)}{\lambda^{1/\alpha}}$, where Γ is the *Gamma function*.

2.2 An analytical approach to marginal renewal costs

The theory behind the approach that we use is developed within a context of structural road repair, but is applicable to any transport mode (Link and Nilsson, 2005). The initial presentation is based on a deterministic model, which later is extended to include stochastic parts. The baseline is that the time span between two renewals is decided by aggregate traffic on a specific segment. Newbery (1988) assumed that the amount of traffic that a road could handle is decided upon in the design phase and hence will affect construction costs. Assuming the prediction of traffic is correct and the sole contributor to deterioration, he introduced the so called *Fundamental theorem*, that is short run marginal cost of road damage equals average cost. Newbery’s theory is extended in Lindberg (2002) who formulates a more general expression for calculation of marginal costs, which we use for the railway track analysis. We will therefore discuss Lindberg’s model within a railway context. Lindberg defines the life time of a track segment as

$$X = \left[\frac{\Theta(q)}{q} \right] e^{-mX} , \qquad (5)$$

where X is the renewal interval, Θ is total tonnage that the track can accommodate between two renewals, q is annual tonnage and m is non-traffic related deterioration. The concept of Lindberg’s approach is that the total tonnage is a function of *actual* annual traffic, rather than being a constant that can be *predicted* in advance as Newbery assumes. Furthermore, non-traffic deterioration can shorten the renewal interval in the form e^{-mX} .

The model assumes that the track has an initial quality of Q^H (figure 1). Traffic volumes reduce this quality over time and a renewal of the track is justified at X^* with quality Q^L, when the initial quality level Q^H is restored. Assuming constant traffic flows, this cycle is repeated into infinity with all future renewal intervals being of length $\overline{X}$. The deterioration of quality over time is associated with a railway track management cost, which can be discounted to any given reference year. The change in costs associated with a marginal increase in traffic (Δq) in a specific year on a specific track segment is of main interest. Following Lindberg (2002), a negative association between q and X is expected, that is more traffic will shorten the renewal interval. The traffic increase at $\tilde{x}$ will affect the quality of the track and shorten the first renewal interval to *X*. Hence, renewal will take place at *X* rather than X^*, and all subsequent renewals will be scheduled earlier than if the increase had not taken place. Discounting and comparing the two alternative cost streams in figure 1 gives the marginal cost associated with the increase in traffic.

[Figure 1]

A schematic view of how the change in traffic affects traffic volumes and renewal intervals is given in figure 2. In all time periods but one during the first renewal interval *X*, an observed traffic volume *q* is assumed (as in *x* in figure 2).

[Figure 2]

We further assume that the increase in traffic occurs in a specific time period, $\tilde{x}$, changing traffic volume from *q* to $q_{\tilde{x}} = q + \Delta q$. The *average* traffic volume in *X* is then defined as $\bar{q}_1 = \frac{(q(X-1) + q_{\tilde{x}})}{X} = q(1 - \frac{1}{X}) + \frac{q_{\tilde{x}}}{X}$. The traffic increase can be viewed as a shock to a system that returns to normal already in the next time period. After the renewal in *X*, traffic volumes of *q* are used as a simplifying assumption in each time period giving constant average traffic flows ($\bar{q}$) and renewal intervals ($\bar{X}$) into infinity.

A track renewal comes at a cost *c* expressed as SEK per track kilometre. The present value of an infinite series of renewals at *X* with subsequent constant intervals $\bar{X}$ can be expressed as

$$PVC_X = c(1 + e^{-r\bar{X}} + e^{-r2\bar{X}} + ... + e^{-rn\bar{X}}), \quad (6)$$

where r is the social discount rate. When n approaches infinity the present value of the renewal cost can be written as

$$\lim_{n\to\infty} PVC_X = c\frac{1}{(1 - e^{-r\bar{X}})} \quad (7)$$

A track segment that is observed in the first renewal interval at time $\tilde{x}$ will have ($X - \tilde{x}$) years remaining before the next renewal occurs. We define the remaining life time $\omega \equiv X - \tilde{x}$, which gives the present value of a track segment analysed in time $\tilde{x}$ as

$$PVC_{\tilde{x}} = ce^{-r\omega}\frac{1}{(1 - e^{-r\bar{X}})}. \quad (8)$$

The present value calculation then consists of two parts where the first is related to the current renewal interval X and the second to all future (constant) intervals $\bar{X}$. As traffic affects the renewal interval, expression (8) is used to calculate the marginal cost, based on the change in present value from a change in traffic. $q_{\tilde{x}}$ is observed annual traffic in $\tilde{x}$ and we take the derivative of $PVC_{\tilde{x}}$ with respect to $q_{\tilde{x}}$. Following Haraldsson (2007),

$$MC_{\tilde{x}} = \frac{\partial PVC_{\tilde{x}}}{\partial q_{\tilde{x}}} = \frac{\partial PVC_{\tilde{x}}}{\partial X}\frac{\partial X}{\partial q_{\tilde{x}}} = -cr\frac{e^{-r\omega}}{(1 - e^{-r\bar{X}})}\frac{\partial X}{\partial q_{\tilde{x}}}. \quad (9)$$

Introducing the concept of a deterioration elasticity, $\varepsilon = \frac{\partial X}{\partial \bar{q}_1}\frac{\bar{q}_1}{X}$, as a measure of the change in the first renewal interval from a percentage change in traffic, we get expression (10) (Haraldsson, 2007), where $\bar{q}_1$ is the average annual traffic volume of the first renewal interval.

$$\frac{\partial X}{\partial q_{\tilde{x}}} = \frac{\partial X}{\partial \bar{q}_1}\frac{\partial \bar{q}_1}{\partial q_{\tilde{x}}} = \left[\frac{\partial \bar{q}_1}{\partial q_{\tilde{x}}} = \frac{1}{X}\right] = \frac{\varepsilon}{\bar{q}_1} \tag{10}$$

(10) in (9) gives

$$MC_{\tilde{x}} = \frac{\partial PVC_{\tilde{x}}}{\partial q_{\tilde{x}}} = -cr\frac{e^{-r\omega}}{(1-e^{-r\bar{X}})}\frac{\varepsilon}{\bar{q}_1}\,. \tag{11}$$

Haraldsson (2007) develops the theoretical foundation in Lindberg (2002) further to situations when renewal intervals are not deterministic, but follow some probability density *g*, positive for positive arguments.

$$E\left[\frac{\partial PVC_{\tilde{x}}}{\partial q_{\tilde{x}}}\right] = -cr\frac{\varepsilon}{\bar{q}_1}\int_0^{\infty}\frac{e^{-r\omega}}{(1-e^{-r\bar{X}})}g(\omega)d\omega \tag{12}$$

Given the survival function *S* for the track life time, the density function for the remaining life time in a renewal process is $g(\omega) = \frac{S(\omega)}{\mu}$, where $\mu = E(X)$ as before (Lancaster, 1990).

Assuming Weibull distributed lifetimes, $g(\omega) = \frac{e^{-\lambda\omega^a}}{\mu}$. This gives expression (13), the expected present value of the marginal cost with respect to tonnage, which has no closed form and must be solved with numerical integration[2].

$$E\left[\frac{\partial PVC_{\tilde{x}}}{\partial q_{\tilde{x}}}\right] = -cr\frac{\varepsilon}{\bar{q}_1}\int_0^\infty \frac{e^{-r\omega}}{(1-e^{-r\bar{X}})}\frac{e^{-\lambda\omega^a}}{\mu}d\omega = -\varepsilon\frac{c}{\bar{q}_1\mu}\frac{r}{(1-e^{-r\bar{X}})}\int_0^\infty e^{-r\omega-\lambda\omega^\alpha}d\omega \qquad (13)$$

The deterioration elasticity (ε) is estimated in the survival model with an accelerated failure time (AFT) Weibull error specification. The additional parameters to be derived from the survival analysis are the expected life time of a track segment (μ), the Weibull scale (λ) and shape (α) parameters. *c* is the cost per kilometre for renewing a railway track, $\bar{q}_1$ is the average annual traffic volume of the first renewal interval and *r* is the social discount rate.

3.0 Railway data

The infrastructure data at hand has been collected from the Swedish National Rail Administration's (Banverket) track information system (*BIS*). *BIS* contains information about homogenous track segments on the main lines in Sweden. Station areas and freight marshalling yards are excluded from the analysis due to difficulties in allocating traffic volumes to individual segments. Each included segment has information on which year the track was laid and some additional technical characteristics of the track as well as organisational identity.

Two data sets from *BIS* are matched. The first is from December, 1999 and the second from December, 2005. Rail renewals between these years are identified through changes in the infrastructure information. From the information on which year the track is laid, we derive

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[2] Note that this is an approximation to a complete stochastic formulation of the marginal cost expression, in which $\bar{X}$ should also be a random variable as future renewal intervals are unknown and stochastic by nature. As *X* also is stochastic, ε is stochastic. This will be developed further in subsequent work.

an age variable for each observation. Any change during the study period results in two observations, one for the initial track that is replaced, which becomes an uncensored observation where we observe the full length of the renewal cycle, and another for the new track that is censored at the end of 2005 where only current age is observed. For an observation with no changes registered, a censored observation is added with age as per the end of 2005[3].

The decision to renew a track segment is assumed to be taken when the track has reached a specific quality target. In real life, such a unique quality target is non-existent, but more likely the decision will be based on several information sources about the quality of the track. The overall objective though is always to minimise life cycle cost. When the discounted life cycle cost of operating and maintaining the old track in the future exceeds the discounted sum of the renewal cost and future track operation and maintenance costs, a renewal is justified. In our context, wear and tear over time from train passages will affect the cost of infrastructure operation and maintenance, and hence decide the optimal timing of a renewal.

Since no comprehensive traffic database exists in Sweden, a time-series of traffic data is created based on various sources of information. Andersson (2006) provides traffic for the period 1999-2002, and from 2002 to 2005, we extrapolate using traffic growth coefficients derived from access charge declarations by train operators to Banverket. This method gives an estimate of annual track segment traffic for the time window 1999-2005. Table 1 gives the basic data descriptive statistics.

[Table 1]

[3] This method has a small risk of missing a renewal interval that is shorter than the time window, but with the long life times that railway tracks have, this risk is negligible.

The original data set consists of 1 631 observations, but missing age and traffic data reduces the number of observations to 1 493, of which 1 333 observations are censored (90.1 per cent). The mean age is 21.6 years, which includes both observed life times and current ages. Approximately 60 per cent of all traffic in gross tonnes is freight traffic and a freight train is more than 4 times heavier than a passenger train on average. 65 per cent of main lines track segments are single tracks and more than 80 per cent have continuously welded rails.

4.0 Estimated survival models

In this section, the results from different model specifications of the survival analysis are presented. The purpose of the survival analysis is to estimate models that provide input into expression (13), namely the deterioration elasticity ε, the scale and shape parameters λ and α as well as the expected life time μ.

The parametric survival model with a Weibull distribution has an accelerated failure time representation as well as a linear representation in the logarithm of time (or age) (Klein and Moeschberger, 2002). Let x be the observed age of observation i and $\mathbf{z}_i$ a vector of covariates fixed over time for the same observation. Cleves et al. (2004) formulates age in the AFT model as $x_i^* = \exp(-\mathbf{z}_i'\boldsymbol{\beta})x_i$ where $x_i^* \sim Weibull(\lambda, \alpha)$. The intuition behind the AFT formulation is that $\exp(-\mathbf{z}_i'\boldsymbol{\beta})$ works as an acceleration factor on x. If the covariates in $\mathbf{z}$ are zero, $x_i = x_i^*$, which is the baseline risk. Every observation faces the same hazard function, but as time goes by, the acceleration factor generated by the individual covariates will affect the passage of time itself. The AFT representation can be rewritten as

$$\ln(x_i) = \mathbf{z}_i'\boldsymbol{\beta} + \ln(x_i^*) = \beta_0 + \mathbf{z}_i'\boldsymbol{\beta} + u_i, \qquad (14)$$

where u_i is extreme value distributed with shape parameter α.

Maximising the following log-likelihood function will give the maximum likelihood estimates of $\boldsymbol{\theta} = (\alpha, \boldsymbol{\beta}, \lambda)$ in the presence of right censored observations (Wooldridge, 2002).

$$\ln L(\boldsymbol{\theta}) = \sum_{i=1}^{N} \left\{ d_i \ln\left[f\left(x_i \mid \mathbf{z}_i; \boldsymbol{\theta}\right)\right] + \left(1 - d_i\right) \ln\left[1 - F\left(x_i \mid \mathbf{z}_i; \boldsymbol{\theta}\right)\right]\right\}, \quad (15)$$

where d_i = 1 for uncensored observations; 0 otherwise, N = number of observations, $\mathbf{z}_i$ = a vector of covariates for observation i, $\boldsymbol{\theta}$ = a vector of unknown model parameters to be estimated. $f()$ is the probability density function and $F()$ is the cumulative distribution function. The second term equals the logarithm of the survival function of the Weibull model and this is the only information that the censored observations provides, the probability of surviving at least to x. Conversely, the uncensored observations provide the unconditional probability of being renewed in time x.

The survival analysis is performed in three stages. In the first stage, the output representation is analysed. Total tonnage per segment is compared to splitting the total into freight and passenger tonnage. A negative relationship between traffic volumes and life times is expected. In the second stage, some features of the infrastructure are added to the analysis, and in the third stage, dummy variables for track district location are included. The statistical software package *Stata*, version 9 (StataCorp, 2005) is used for all model estimations.

The different specifications are evaluated with a *link test*. Cleves et al. (2004) recommends this test as a search for variables to add to the model and it is based on the following steps. Estimate the model and predict the outcomes from the model. Generate the square of the predictions and re-estimate the model using the predictions and their squares as explanatory variables. If the initial model is correctly specified, the coefficient for the squared predictions is insignificantly different from zero.

A likelihood ratio test (*LR*) is performed for the specified models versus a constant only model. Furthermore, Akaike's (*AIC*) and Bayes Information Criteria (*BIC*) are also used. These criteria penalise complex models by adding a factor to the traditional log likelihood calculation as the number of model parameters are increased. The general formula for these criteria is $IC = -2\log(\hat{L}) + \gamma K$ where *K* is the number of free parameters in the model. The difference between the two versions is that the AIC uses γ = 2 and BIC uses γ = log(*N*) where *N* is the number of observations in the model estimation. The BIC puts a heavier penalty on the log likelihood than the AIC for large samples.

The last model specification test is to plot the *Cox-Snell* (pseudo) residuals against the cumulative hazard function *H*(*x*) as recommended by Klein and Moeschberger (2002). In a correct model specification, the residuals follow a 45 degree reference line. Strong deviations from this line indicate a misspecified model.

All model coefficients are expressed in AFT format. 1 493 observations are used in the estimations and 90.1 per cent are censored. The results of the models in stage 1 are given in table 2.

[Table 2]

The coefficient for log output in *Model 1* has the expected negative sign and is significant at the 5 per cent level. Since *Model 2* includes squared terms, we have to evaluate the model to derive the elasticities and they are also significantly negative at the same level of risk. Terms of higher order have been tried for both models, but these are insignificant in *Model 1*. The Weibull shape parameter is close to 3.2 in both specifications indicating strong, positive duration dependence (increasing risk over time) as well as a more than proportional acceleration of the renewal risk with respect to accumulated traffic.

The LR test favours both models before a constant only model, but the link test indicates a misspecification of *Model 2* as the term for the squared predictions is significantly different from zero. Furthermore, both the AIC and BIC are in favour of *Model 1*. Plotting the Cox-Snell residuals for both models indicate a better fit though for *Model 2*, the right graph in figure 3.

[Figure 3]

The outcome of stage 1 is that our tests are inconclusive in choosing between the two specifications. We expect that not only traffic will affect the renewal times, but more likely a combination of traffic and infrastructure characteristics. In the second stage, we extend the first models to also include the infrastructure. Effects from the continuous variable track segment length and dummy variables for continuous welded rails, rails below 50 kg per metre and single tracks are studied. The results are given in table 3. The only significant infrastructure variable is the dummy that separates single tracks from multiple tracks, independent of whether we represent traffic as a total or split into freight and passenger. The negative sign is expected as single tracks have a shorter renewal cycle given that the traffic loads cannot be distributed over several tracks. The coefficient for total gross tonnes drops from -0.116 (*Model 1*) to -0.188 (*Model 3a*), but the 95 per cent confidence intervals (*CI*) of the point estimates overlap. The model suggests that when controlling for traffic and single/multiple tracks, there are no significant differences between track segments of different length, continuous welded rails or older tracks with rails below 50 kg per metre.

We therefore re-estimate reduced models including only traffic and the dummy for single tracks (*3b* and *4b*). The coefficients for our traffic variables are in the same range as *Model 3a* and *4a*. The estimated shape parameter (α) remains around 3.2 for all models.

The AIC and BIC point in opposite directions and the plot of the Cox-Snell residuals (figure 4) once again indicates a better fit of the model with split traffic to the data. Link tests show an insignificant (at 5 per cent level) squared term for *Model 3b* predictions, while the test for *4b* just passes this risk level.

[Figure 4]

In the final stage, we also include dummy variables for track district location. Previous analyses of cost data for maintenance and renewal (Andersson, 2006) indicate that these effects have a role in explaining cost variations over the Swedish railway network.

The model estimates from the third and final stage are given in table 4. The traffic variables remain significant with expected signs and in the range of the models with only the dummy for single tracks. Only a few of the track district dummy variables turn out significant, indicating small regional differences. The estimate of the shape parameter increases from 3.2 to 3.5.

Since a regression model in logarithmic form is used, the coefficients can be interpreted as elasticities (Gujarati, 1995). The coefficient for total gross tonnes in *Model 5* is the deterioration elasticity with a point estimate of -0.126. A percentage increase in traffic would lead to a 0.13 per cent reduction in expected rail life, ceteris paribus. Since *Model 6* includes a squared term, the elasticity is not constant over the range of traffic as in *Model 5*.

[Table 4]

Expression (16) is used to calculate track segment specific deterioration elasticities (*standard error*), $\hat{\varepsilon}_i$, of *Model 6* and predicted elasticities are given in figure 5.

$$\hat{\varepsilon}_i = \frac{\partial \ln X}{\partial \ln q_i} = \hat{\beta}_{\ln q} + 2\hat{\beta}_{(\ln q)^2} \ln q_i \qquad (16)$$

The mean of these elasticities ($\bar{\varepsilon}$) are for freight traffic -0.129 (*0.0318*) and for passenger traffic -0.092 (*0.0442*). Hence, a percentage increase in passenger traffic reduces rail life on average by 0.092 per cent while the reduction from freight is 0.129 per cent. The confidence intervals though are too wide to claim the elasticities significantly different from each other at the 5 per cent level.

[Figure 5]

Finally, the hazard and survival functions of *Model 6* are presented in figure 6. The survival probability is quite high for the first 25 years, but is then reduced in an accelerating manner. Unless maintenance activities fully compensate for wear, tear and climate effects, this pattern is expected, which is also represented by increasing hazard rates over time.

Both the AIC and BIC are in favour of *Model 6*. The link tests show insignificant squared terms for both models and a plot of the Cox-Snell residuals indicate that the model with split traffic seems to perform slightly better, but with a little drift in the residuals in the top right corner (figure 7, right), which is not an uncommon result.

[Figure 6]

[Figure 7]

Assuming that the data is Weibull distributed with a high share of censored observations can be crucial. To check for sensitivity, we have estimated a Cox proportional hazard model, which is semi-parametric and free of assumptions on the underlying risk structure. The estimates of the Cox model are compared with the estimates of a proportional hazard representation of the Weibull model. We find no major differences in these estimates and hence conclude that the Weibull assumption is valid.

5.0 Marginal costs

The specification tests in the end of section 4 favour *Model 6*. Expression (17) follows (13) and is used for the marginal cost calculation. It contains four distinct parts. The first part is the deterioration elasticity, which is estimated in our survival models. The second part is the average cost of a rail renewal. Note that total traffic has to be used in the denominator to get the correct average cost calculation. Using freight and passenger traffic separate in this part would lead to an overstatement of the cost. The third part is the discount factor of an infinite cycle of estimated average life times $\bar{X}$. The fourth part adjusts the calculation to the distribution of rail ages and remaining expected life times in our sample. This part has no closed form and is solved by numerical integration. We limit the integration area to 0 - 100, as we have no observed rails over 80 years in our sample. A test with 200 years has been done without significant impact on the marginal cost estimates.

$$E\left[\frac{\partial PVC_{\tilde{x}}}{\partial q_{\tilde{x}}}\right] = -\hat{\varepsilon}_{ij}\,\frac{c}{\bar{q}_1\hat{\mu}_i}\,\frac{r}{(1-e^{-r\bar{X}})}\int_0^{\infty} e^{-r\omega-\hat{\lambda}_i\omega^{\hat{\alpha}}}\,d\tilde{x} \tag{17}$$

Since squared terms are included in *Model 6*, the elasticities are non-constant, and depend on tonnage. We therefore calculate the elasticity for each observation *i* and traffic category *j* in our sample. *c* is the track renewal cost and Banverket estimates this cost on average to SEK 4 500 000 per kilometre track. *r* is the social discount rate, which in Swedish public transport infrastructure projects is set to 4 per cent. The predicted life time for an observation, is calculated as $\hat{\mu}_i = \exp(\mathbf{z}_i'\boldsymbol{\beta})$. $\hat{\alpha}$ is the shape parameter for the Weibull distribution, estimated to 3.506 for *Model 6*. The scale parameter $\hat{\lambda}_i$ is observation specific and is calculated as $\hat{\lambda}_i(x \mid \mathbf{z}_i) = \hat{\mu}_i^{-\hat{\alpha}}$. Table 5 summarises the marginal cost estimates for freight and passenger trains.

[Table 5]

A normal mean value of the individual marginal costs for each traffic category gives negative point estimates. This anomaly comes from some of the deterioration elasticities being positive for very low traffic volumes. Since the average cost is high at low traffic volumes, these segments generate high negative marginal costs. Using a simple mean value in a pricing scheme, would place too much weight on low volume observations. In previous studies, Andersson (2006, 2007) handles this by placing different weights on the track segment specific marginal costs in accordance with the segments share of total gross tonne kilometres. This is a revenue-neutral scaling procedure, and the weighted estimates for freight trains are given in table 5, row 2. The impact from the negative marginal costs is then reduced.

Another solution to the problem is to exclude observations with negative marginal costs entirely. This gives an overall positive marginal cost estimate for freight. However, as seen in figure 8, the marginal cost still drops sharply from low to high levels of output. Using the same scaling procedure as above, the weighted marginal cost estimate is in line with the full sample estimate.

[Figure 8]

The same calculations are done for passenger trains and are given in table 5, (rows 5-8) and we observe the same pattern. A plot of the positive marginal cost estimates are given in figure 9. Note that passenger trains generate a higher weighted marginal cost than freight trains, in the region of SEK 0.0025-30 per gross tonne kilometre, despite having a lower point estimate of the deterioration elasticity. This comes from a slightly different distribution of individual elasticities and average cost estimates.

[Figure 9]

6.0 Discussion and conclusions

Pricing at marginal cost for railway use is important from an efficiency standpoint. In this paper, we have studied the overall issue of wear and tear, and specifically the issue of marginal costs related to rail renewal. In contrast to previous cost function efforts, a method that calculates the difference in the present value of rail renewal costs related to changes in rail renewal cycles from different levels of traffic is used. A crucial factor in this calculation is to estimate the *deterioration elasticity*, the percentage change in rail life from a percentage change in traffic. This is done using survival analysis in the form of an accelerated failure time Weibull model. Several specifications of the Weibull model are analysed and we test for potential errors in the specifications. A model that splits total traffic into freight and passenger tonnage is recommended and the estimated deterioration elasticity is slightly higher for freight trains than for passenger trains. We estimate the marginal cost to approximately SEK 0.0020 per freight gross tonne kilometre and SEK 0.0025-30 per passenger gross tonne kilometre. Somewhat surprising, marginal cost for passenger trains are slightly higher than for freight trains. From a deterioration perspective, this is not expected, but what is observed in the data is a rail renewal pattern expressed as deterioration elasticities driven by passenger trains more than freight trains. One explanation to this can be the higher demands for good quality tracks by high speed passenger trains, which results in shorter renewal intervals than would be the case if the track served only freight trains, especially at high traffic volumes.

There are some points that should be emphasised. The first is that these estimates are very similar to previous econometric cost function estimates. Andersson (2006) applies pooled ordinary least squares (POLS) to track section cost data and finds that the *increase* in marginal cost, when renewals are added to maintenance costs, is SEK 0.0024 per gross tonne kilometre. One has to be aware of the different cost bases used in this study and Andersson (2006) though. Here, only track renewal costs are used, while Andersson (2006) uses all infrastructure renewal costs.

There seems to be evidence for a price relevant rail renewal cost in Sweden, which should be included in future pricing schemes if marginal cost pricing is aimed at. The size of such a wear and tear fee for rail renewal depends on how the actual pricing scheme is designed, either using a flat rate for all trains or separate rates for freight and passenger trains. We are looking at a fee around SEK 0.0025 per gross tonne kilometre. The latest *Network Statement* by Banverket (Banverket, 2006) holds the official wear and tear fee for 2007, which is SEK 0.0029 per gross tonne kilometre. The latest revision of this fee is based on econometric analyses of maintenance costs in Andersson (2006). Adding a fee for renewals would double the fees payable for wear and tear by train operators in Sweden. Furthermore, Andersson (2007) has estimated the marginal cost for wear and tear using fixed effects models in the range of SEK 0.007 per gross tonne kilometre indicating that railway infrastructure charges in Sweden today are well below marginal cost.

The second point is that the method as such is working, which can be an opening for marginal cost estimates when a time-series of renewal cost data is not at hand. There are two main sources of information that are necessary in this case. One is a well functioning track information system and the other is a traffic information system. It is important to point out that this information is needed for a few years only, during which at least some renewals have been undertaken. Modern railway track information systems have the possibility to record not only the current status of the network at a detailed level, but also its previous status. This will, over time, generate a rich information source for this type of analysis and as we add more uncensored observations to the database, we will hopefully increase precision in our estimates.

An obvious field of development is to make a good representation of the infrastructure variables. We have not been able to identify different types of tracks being of importance for our estimates. One reason for this can be that the underlying spending pattern on maintenance during the life cycle is adjusted in such a way that the inherent quality differences between tracks are levelled out. This is an area where more work is needed in the future, and where an extended database will provide possibilities for some answers.

The marginal cost is highly dependent on and directly proportional to the cost estimate for rail renewals given by Banverket. To get a better estimate, we need to look closer at rail renewals of different types to be able to predict the rail renewal cost at the track segment level.

On the theoretical representation, introducing a stochastic representation of future renewal intervals could be the next area to develop. We are currently assuming all future intervals being equal, which most likely is too strong. Whether this would have a substantial effect on the marginal cost estimates is difficult to say. Since we use a discounting procedure, future costs are of less importance.

References

Andersson, M. (2006): 'Marginal Cost Pricing of Railway Infrastructure Operation, Maintenance and Renewal in Sweden: From Policy to Practice Through Existing Data', *Transportation Research Record: Journal of the Transportation Research Board*, 1943, 1-11.

Andersson, M. (2007): 'Fixed Effects Estimation of Marginal Railway Infrastructure Costs in Sweden', Chapter 3 in Andersson, M. (2007) *Empirical Essays on Railway Infrastructure Costs in Sweden*. Ph.D. Thesis. ISBN: 91-85539-18-X. Division of Transport and Location Analysis, Department of Transport and Economics, Royal Institute of Technology, Stockholm, Sweden.

Banverket (2006): *Network Statement*, 2006-12-10--2007-12-08(T07), Banverket, Market Division, Borlänge, Sweden.

Bruzelius, N. (2004): 'Measuring the Marginal Cost of Road Use – An International Survey', *VTI Meddelande 963A*, Linköping, Sweden.

Cleves, M. A., W. W. Gould and R. G. Gutierrez (2004): *An Introduction to Survival Analysis Using Stata*, Stata Press, College Station, Tx.

Gujarati, D. N. (1995): *Basic Econometrics*, 3rd ed., McGraw-Hill, Singapore.

Haraldsson, M. (2007): 'The Marginal Cost for Structural Repair of Roads - A Duration Analysis Approach', Mimeo. Transport Economics Unit, Swedish National Road and Transport Research Institute, Stockholm, Sweden.

Kiefer, N. M. (1988): 'Economic Duration Data and Hazard Functions', *Journal of Economic Literature*, 26, 646-679.

Klein, J. P. and M. L. Moeschberger (2002): *Survival Analysis - Techniques for Censored and Truncated Data*, 2nd ed., Springer, New York, NY.

Lancaster, T. (1990): *Econometric Analysis of Transition Data*, Cambridge University Press, Cambridge, UK.

Lindberg, G. (2002): 'Marginal Cost of Road Maintenance for Heavy Goods Vehicles on Swedish Roads', *Annex A2, Deliverable 10: Case Studies on Marginal Infrastructure Costs*, UNITE (UNIfication of accounts and marginal costs for Transport Efficiency), Funded by 5th Framework RTD Programme, Institute for Transport Studies, University of Leeds, Leeds, UK.

Link, H. and J.-E. Nilsson (2005): 'Infrastructure', in C. Nash and B. Matthews (eds.) *Measuring the Marginal Social Cost of Transport, Research in Transportation Economics Vol. 14*, Elsevier, Oxford, UK, ch. 3.

Nash, C. and T. Sansom (2001): 'Pricing European Transport Systems: Recent Developments and Evidence from Case Studies', *Journal of Transport Economics and Policy*, 35, 363-380.

Nash, C. (2005): ‘Rail Infrastructure Charges in Europe’, *Journal of Transport Economics and Policy*, 39, 259-278.

Nash, C. (2003) UNITE (UNIfication of accounts and marginal costs for Transport Efficiency), Final Report for Publication, Funded by 5th Framework RTD Programme, Institute for Transport Studies, University of Leeds, Leeds, UK.

Nash, C. and B. Matthews (eds.) (2005): *Measuring the Marginal Social Cost of Transport, Research in Transportation Economics Vol. 14*, Elsevier, Oxford, UK.

Newbery, D. M. (1988): ‘Road User Charges in Britain’, *The Economic Journal*, 98, 161-176.

StataCorp. (2005): *Stata Statistical Software: Release 9*. College Station, Tx.

Thomas, J., F. Dionori and A. Foster (2003): ‘EU Task Force on Rail Infrastructure Charging. Summary Findings on Best Practice in Marginal Cost Pricing’, *European Journal of Transport and Infrastructure Research*, 3, 415-431.

Wooldridge, J. M. (2002): *Econometric Analysis of Cross Section and Panel Data*. MIT Press, Cambridge, Mass.

Figure 1. Renewal intervals with and without a marginal increase in traffic at $\tilde{x}$

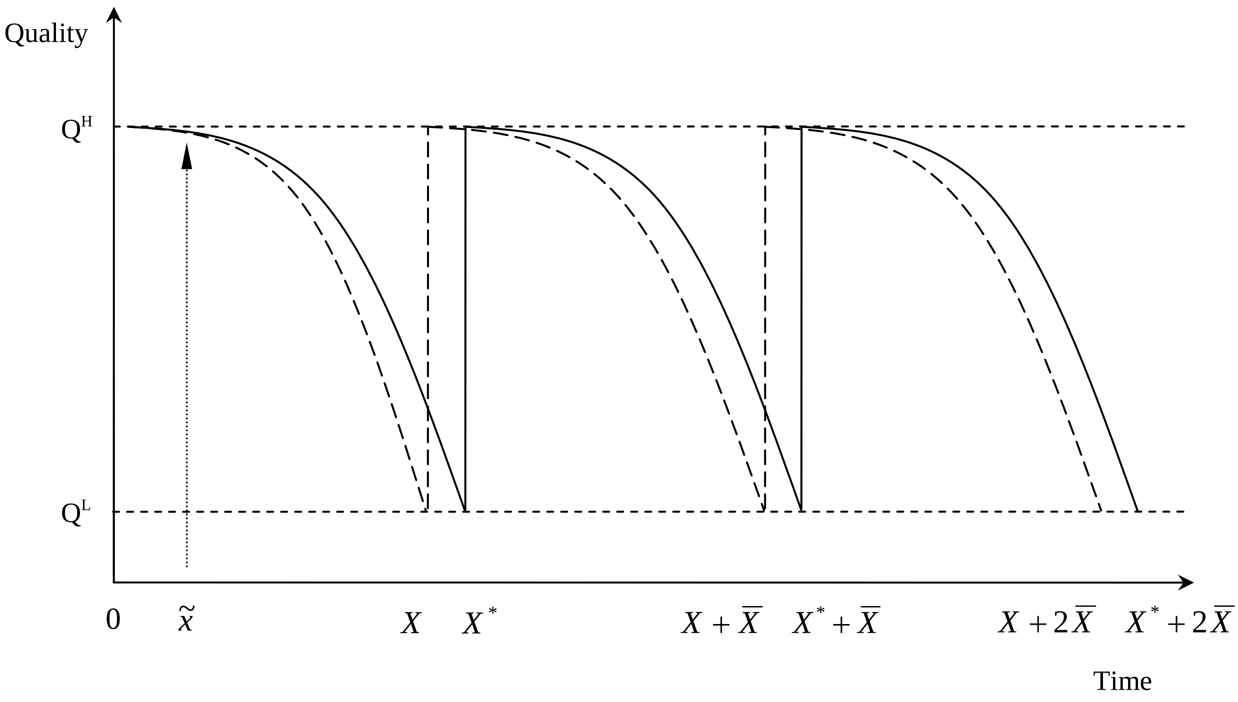

Figure 2. Traffic volumes and renewal intervals from a marginal increase at $\tilde{x}$

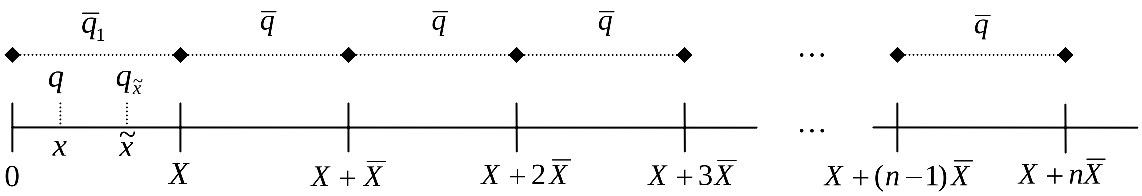

Figure 3. Cox-Snell (C-S) residual specification test for *Model 1* and *2*

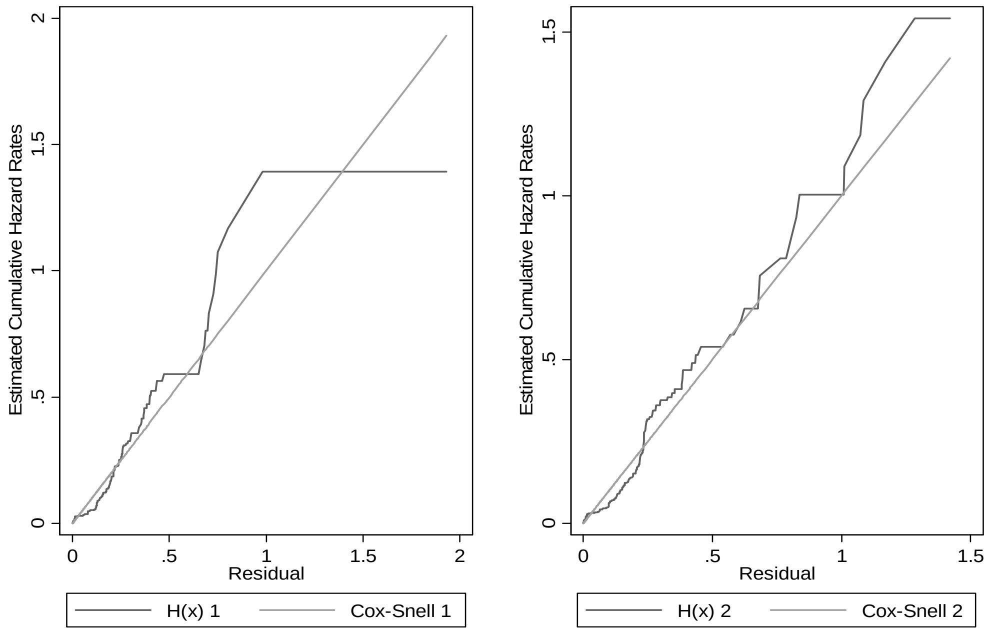

Figure 4. Cox-Snell (C-S) residual specification test for *Model 3b* and *4b*

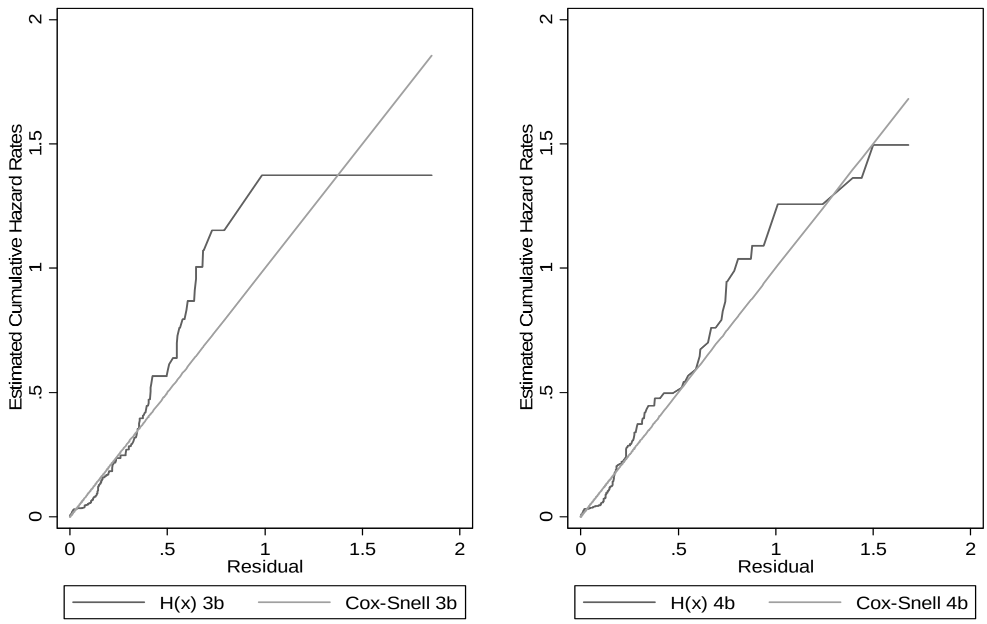

Figure 5. Predicted freight and passenger traffic deterioration elasticities for *Model 6*

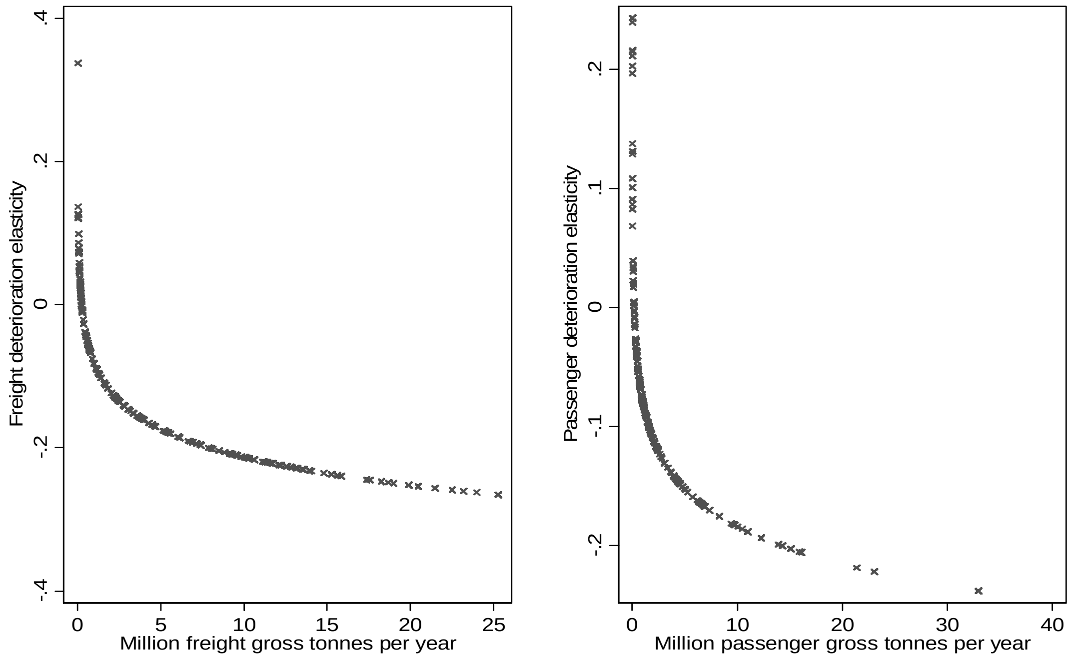

Figure 6. The survival and hazard functions for *Model 6*

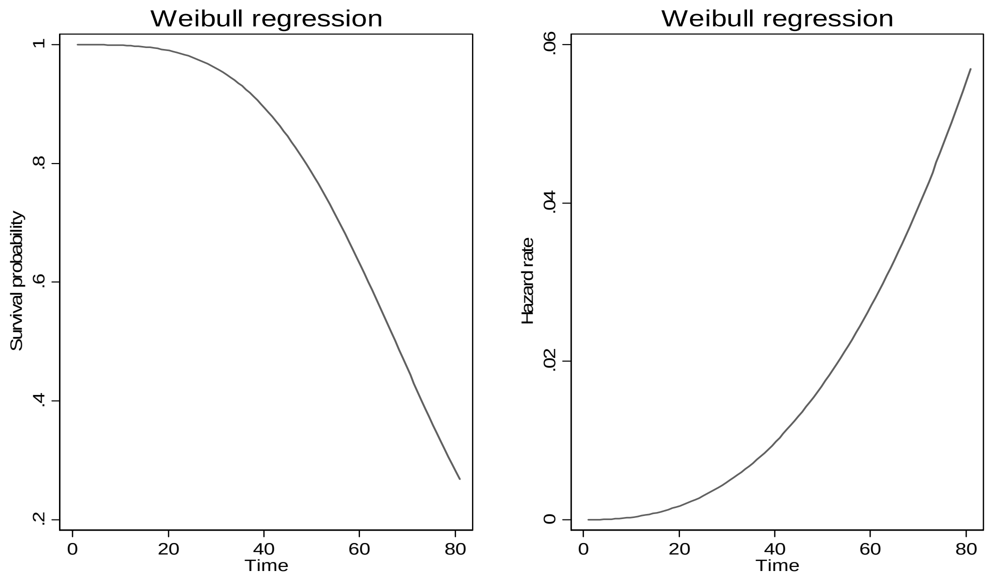

Figure 7. Cox-Snell (C-S) residual specification test for *Model 5* and *6*

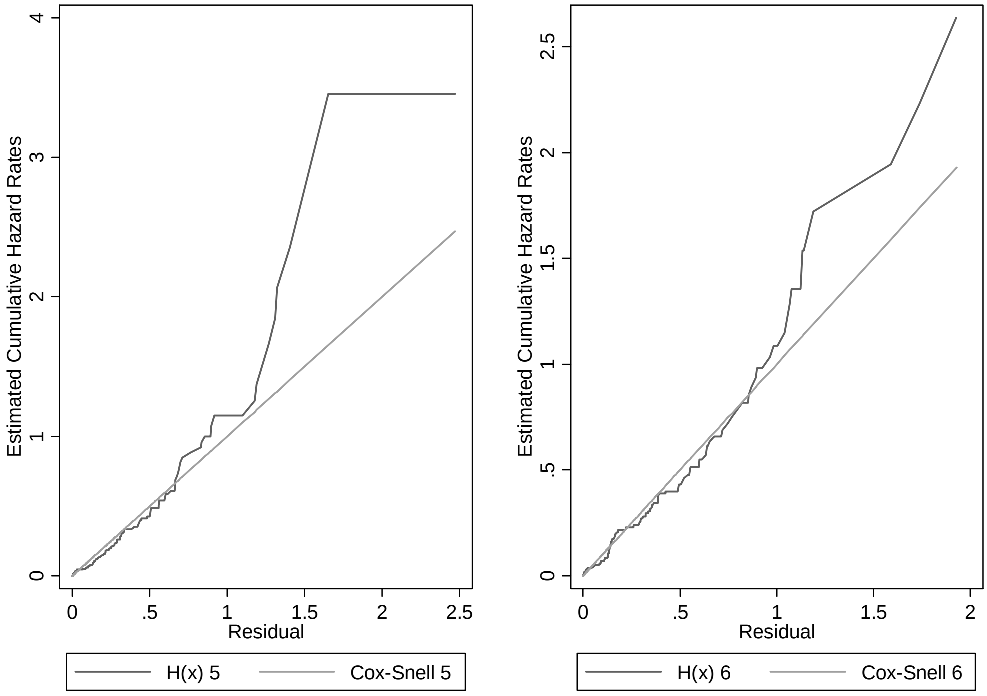

Figure 8. Track segment marginal costs - Freight trains, reduced sample

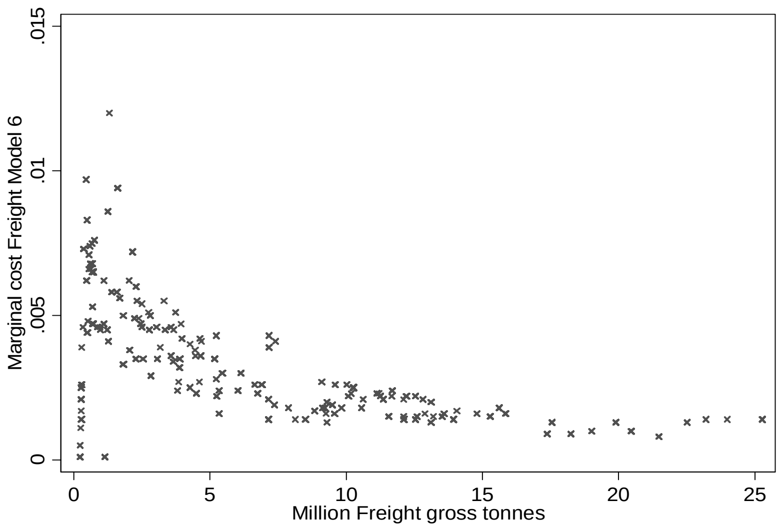

Figure 9. Track segment marginal costs - Passenger trains, reduced sample

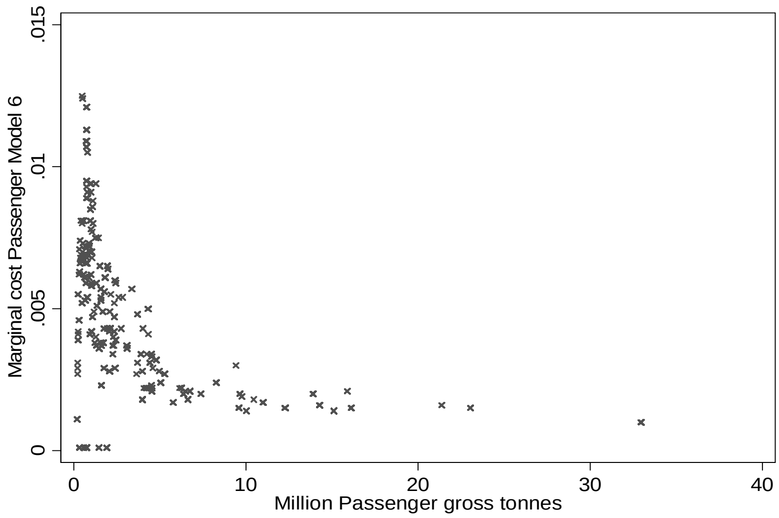

Table 1. Descriptive statistics of the data set

Variable	Mean	Std. Dev.	Min	Max
Age (years)	21.6	13.5	1	81
Track segment length (metres)	6 134	3 971	1	24 899
Total traffic volume (gross tonnes)	9 854 735	9 004 532	104 661	34 558 100
Total trains (number of)	21 323	23 977	319	122 838
Average train weight (gross tonnes)	528	398	52	2 631
Freight traffic volume (gross tonnes)	6 006 180	6 320 835	557	25 270 000
Freight trains (number of)	5 238	5 020	2	20 082
Average freight train weight (gross tonnes)	959	413	171	3 177
Passenger traffic volume (gross tonnes)	3 848 553	6 143 928	800	32 957 900
Passenger trains (number of)	16 085	22 971	3	121 305
Average passenger train weight (gross tonnes)	228	106	41	694
Dummy - Single tracks	0.646	0.478	0	1
Dummy - Rails <50 kg	0.143	0.350	0	1
Dummy - Continuously Welded Rails	0.803	0.398	0	1
Dummy - Track District Borlänge	0.058	0.234	0	1
Dummy - Track District Stockholm	0.092	0.290	0	1
Dummy - Track District Falköping	0.072	0.258	0	1
Dummy - Track District Göteborg	0.077	0.267	0	1
Dummy - Track District Gävle	0.073	0.260	0	1
Dummy - Track District Hallsberg	0.056	0.229	0	1
Dummy - Track District Hässleholm	0.065	0.247	0	1
Dummy - Track District Kiruna	0.031	0.175	0	1
Dummy - Track District Karlstad	0.073	0.260	0	1
Dummy - Track District Luleå	0.017	0.131	0	1
Dummy - Track District Malmö	0.089	0.285	0	1
Dummy - Track District Nässjö	0.067	0.250	0	1
Dummy - Track District Norrköping	0.046	0.210	0	1
Dummy - Track District Umeå	0.038	0.192	0	1
Dummy - Track District Västerås	0.058	0.233	0	1
Dummy - Track District Ånge	0.087	0.282	0	1
Indicator - Censored Observations	0.901	0.299	0	1

Sources: Banverket Track Information System (BIS) and Andersson (2006)

Table 2: Weibull AFT models with traffic variables only

Variable	***Model 1***		***Model 2***	
	Coeff.	**(S.E.)**	**Coeff.**	**(S.E.)**
Constant	5.815‡	(0.2617)	-0.084	(1.0704)
ln*Total Gross Tonnes*	-0.116‡	(0.0174)	-	
ln*Freight Gross Tonnes*	-		0.439‡	(0.1312)
(ln*Freight Gross Tonnes*)^2	-		-0.018‡	(0.0049)
ln*Passenger Gross Tonnes*	-		0.280‡	(0.1035)
(ln*Passenger Gross Tonnes*)^2	-		-0.011†	(0.0044)
α	3.237	(0.1839)	3.219	(0.1886)
Log likelihood		-331.87		-329.61
Likelihood Ratio χ^2		37.15 (1 df)		41.68 (4 df)
Number of observations		1 493		1 493
AIC		669.74		671.21
BIC		685.67		703.06

‡ Significant at 1 per cent level; † Significant at 5 per cent level; * Significant at 10 per cent level.

Table 3. Weibull AFT models with traffic and infrastructure variables

Variable	***Model 3a***		***Model 4a***		***Model 3b***		***Model 4b***	
	Coeff.	**(S.E.)**	**Coeff.**	**(S.E.)**	**Coeff.**	**(S.E.)**	**Coeff.**	**(S.E.)**
Constant	7.187‡	(0.7548)	-1.193	(1.0276)	6.590‡	(0.3562)	-1.396	(0.9803)
ln*Total Gross Tonnes*	-0.188‡	(0.0440)		-	-0.154‡	(0.0209)	-	
ln*Freight Gross Tonnes*	-		0.474‡	(0.1153)	-		0.483‡	(0.1141)
(ln*Freight Gross Tonnes*)^2	-		-0.020‡	(0.0046)	-		-0.020‡	(0.0043)
ln*Passenger Gross Tonnes*	-		0.591‡	(0.1142)	-		0.601‡	(0.1139)
(ln*Passenger Gross Tonnes*)^2	-		-0.026‡	(0.0052)	-		-0.026‡	(0.0050)
ln*Track segment length*	-0.031	(0.0349)	-0.035	(0.0346)	-		-	
D - Continuous welded rails	0.241	(0.1473)	0.169	(0.1507)	-		-	
D - Rails <50 kg	0.134	(0.1358)	0.187	(0.1341)	-		-	
D - Single tracks	-0.251‡	(0.0849)	-0.461‡	(0.1102)	-0.248‡	(0.0801)	-0.484‡	(0.1028)
α	3.181	(0.1917)	3.276	(0.1989)	3.214	(0.1838)	3.246	(0.1877)
Log likelihood		-324.87		-316.13		-326.53		-317.52
Likelihood Ratio χ^2		51.16 (5 df)		68.64 (8 df)		47.83 (2 df)		65.84 (5 df)
Number of observations		1 493		1 493		1 493		1 493
AIC		663.74		652.25		661.06		649.05
BIC		700.90		705.34		682.30		686.21

‡ Significant at 1 per cent level; † Significant at 5 per cent level; * Significant at 10 per cent level. *D* – Dummy variable

Table 4. Weibull AFT models with traffic, infrastructure variables and track district dummies

Variable	***Model 5*** **Coeff.**	**(S.E.)**	***Model 6*** **Coeff.**	**(S.E.)**
Constant	6.045‡	(0.4774)	-2.860†	(1.2558)
ln*Total Gross Tonnes*	-0.126‡	(0.0262)	-	
ln*Freight Gross Tonnes*	-		0.692‡	(0.1655)
(ln*Freight Gross Tonnes*)^2	-		-0.028‡	(0.0065)
ln*Passenger Gross Tonnes*	-		0.545‡	(0.1518)
(ln*Passenger Gross Tonnes*)^2	-		-0.023‡	(0.0067)
D - Single tracks	-0.246‡	(0.0904)	-0.435‡	(0.1133)
D - Track district Borlänge	0.226*	(0.1219)	0.329†	(0.1480)
D - Track district Stockholm	-0.334‡	(0.1157)	-0.446†	(0.2159)
D - Track district Falköping	0.122	(0.1171)	0.073	(0.1224)
D - Track district Göteborg	0.081	(0.1097)	-0.020	(0.1165)
D - Track district Gävle	0.266*	(0.1369)	0.197	(0.1377)
D - Track district Hallsberg	-0.044	(0.1179)	-0.048	(0.1194)
D - Track district Hässleholm	0.242	(0.1593)	0.394*	(0.2213)
D - Track district Kiruna	-0.052	(0.1799)	-0.049	(0.1854)
D - Track district Karlstad	-0.034	(0.1056)	-0.048	(0.1110)
D - Track district Luleå	5.000	(930.49)	4.699	(635.72)
D - Track district Malmö	0.378‡	(0.1207)	0.327†	(0.1349)
D - Track district Nässjö	-0.029	(0.1175)	-0.049	(0.1205)
D - Track district Norrköping	4.761	(408.14)	4.497	(285.62)
D - Track district Umeå	-0.133	(0.1036)	-0.165	(0.1076)
D - Track district Västerås	-0.059	(0.1274)	-0.170	(0.1402)
α	3.484	(0.2141)	3.506	(0.2169)
Log likelihood		-296.18		-283.51
Likelihood Ratio χ^2		108.54		133.87
Number of observations		1 493		1 493
AIC		630.36		611.03
BIC		731.22		727.82

‡ Significant at 1 per cent level; † Significant at 5 per cent level; * Significant at 10 per cent level.
D - Dummy variable.

Table 5. Marginal cost estimates from *Model 6*

Estimate	**Marginal cost***	**Standard error**	**Observations**
Freight - Full sample	-0.1150	0.075647	1 493
Freight - Full sample, Weighted	0.0019	0.000058	1 493
Freight - Reduced sample	0.0034	0.000064	1 234
Freight - Reduced sample, Weighted	0.0020	0.000050	1 234
Passenger - Full sample	-0.4450	0.086812	1 493
Passenger - Full sample, Weighted	0.0025	0.000096	1 493
Passenger - Reduced sample	0.0046	0.000082	1 309
Passenger - Reduced sample, Weighted	0.0030	0.000064	1 309

* SEK/Gross tonne kilometre