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Keywords: Laboratory Experiment, Multi-Unit Auction, Common Value Auction

JEL Codes: C91, C72, D44

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Abstract

This study addresses a discrete common value environment with independent (one-dimensional) private signals, where the seller offers two identical units and the buyers have (flat) demand for both. Each session is conducted with 2, 3 or 4 buyers. Three auction formats are used: the discriminatory, uniform and Vickrey auctions which are all subjected to a variation in the number of bidders and to repeating bid rounds on each subject. The main findings are that there are no significant differences between the uniform and the discriminatory auction in collecting revenue, while the Vickrey auction comes out as inferior. More bidders in the auction result in a greater revenue and level out the performance across the mechanisms. Demand reduction is visible in the experiment, but it is not as prominent as anticipated. Moreover, subjects come closer to equilibrium play over time. Finally, the winner's curse is less severe than what is reported for inexperienced bidders in other studies.

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1 Introduction

Common value (CV) auctions with single unit demand have been studied for quite some time, both theoretically and in the laboratory. The main focus of the experimental research on CV auctions has been on the winner's curse problem, that is, the adverse selection effect produced by a win if not accounted for. But research on multi-unit demand is scarce. The winner's curse problem has not been addressed in this literature; the emphasis in both theoretical and experimental research, when the items for sale are substitutes, has been on demand reduction.[1] A phenomenon in some auction formats is that bidders have an incentive to reduce the demand for units other than the first, since these bids may become the market clearing price. It is found that demand reduction leads to substantial revenue losses for the sellers. There is also a literature concerning mechanism design issues, complementariness and synergies between items, and the role of package bidding (see, for example, Kagel and Levin (2011) for a start in these areas).

The prevalent static multi-unit auction formats in the literature are the discriminatory, the uniform, and the Vickrey auction. The first two formats are those used in the field, whereas the last is never used due to its (allegedly) complicated nature, even though it has nice demand-revealing properties; see, for example, Rothkopf et al. (1990). When we add the common value environment, the ranking of these auction formats in term of revenues becomes an open question. There is also an ongoing discussion in the market for treasury bonds, as well as in the markets for CO_2 allowances, on which of the first two formats above should be used. (Back and Zender (1993) summarize this debate in the independent private value (IPV) case.)

This study features a discrete auction, in the sense that the values of the unit and bidding are only allowed in integer numbers with independent (one-dimensional) private signals, where the seller offers two identical units and the buyers have demand for both. The three auction formats (discussed in the above paragraph) are tried and subjected to a variation in the number of bidders and to repeating bid rounds (15 - 20 rounds) on each subject. Five main questions are scrutinized. (i) which auction format gives the greatest revenue?; (ii) how does the number of bidders affect revenue?; (iii) is there demand reduction in the uniform and Vickrey auctions?; (iv) what are the implications of repeating the auction several rounds on the subjects, that is do we see any learning effects?; and (v), is there a winner's curse, that is do bidders ignore the informational content inherent in winning, and bid too high?

Starting with revenue, we find that the Vickrey auction always gives the least overall revenue, especially in small group sizes. The uniform and the discriminatory auctions run a close race and cannot be separated. This was quite unexpected due to the non-expected result in 2-player groups. (The hypothesis for the uniform auction is that, in 2-player groups, the subjects play more according to the extreme demand reduction prediction. But, in general, they do not.) For large group sizes, the difference in revenue between the Vickrey and the other two formats disappears completely. The answer to the second

[1] Even though Vickrey (1961) was the first to point out the inefficiency of multi-unit auctions in general, Ausubel (2004) and Ausubel and Crampton (2002) emphasized, in common value settings, that the inefficiencies are due to demand reduction.

question is that the more bidders in an auction, the larger is the revenue for the seller. Third, we see demand reduction, but we do not see any extreme demand reduction at all, that is, zero bidding on the second unit. Fourth, we find that subjects do learn to play equilibrium strategies in the course of the game, at least in the discriminatory auction. Moreover, they continue to learn until the final rounds.

For the last question, we find that the winner's curse (WC) is highly present; mostly in the uniform and discriminatory auctions, but also in the Vickrey auction. We distinguish between bidding above the conditional expected value (of winning) up to the naive expected value and above the naive expected value. It is twice as common to bid in the first interval, which (partly) indicates that subjects have difficulties in understanding the winner's curse.

The theoretical model in this article emanates from Ahlberg (2009) where it is presented more thoroughly. There is little earlier theoretical and experimental work on multi-unit demand common value auctions against which to directly compare our results, except for the theoretical article from Álvares and Mazón (2010). They have a theoretical model similar to ours in a continuous setting. Much of the theory that exists focuses on independent private value (IPV) settings, or, to some extent, interdependent value settings. The contribution of the present study to the experimental literature is a different common value generation, which is made somewhat simpler for ease of understanding. There is reason to believe that subjects do not understand the concept of the winner's curse, and overbid as a result. Second, we want to contribute to the ongoing debate on which of the static auction formats one should use in practice, when the value of the object(s) is common to all bidders.

The rest of the paper is organized as follows: Section 2 focuses on earlier research in relation to the stated questions, section 3 presents the theory and the hypotheses, and section 4 outlines the experimental design. Section 5 contains the results, section 6 discusses them and section 7 concludes the study. All computations are found in the Appendix.

2 Earlier Research

In previous analytical research, Ausubel and Crampton (2002) have shown that, in the interdependent value case where the item for sale is infinitely divisible, in many cases, the discriminatory auction outperforms the uniform price auction but, in general, the revenue ranking between the two is ambiguous. In an IPV setting, Engelmann and Grimm (2009) investigate the three auction mechanisms described above and two open counterparts; the open uniform auction and the Ausubel auction, which is a dynamic Vickrey auction. They find that the revenues are greater if a sealed-bid format is used as compared to an open auction; the revenues depend less on which pricing rule is employed.

Kagel and Levin (2001) theoretically predict that as the number of bidders increases, demand reduction will diminish. They confirm this behavior asymmetrically in a laboratory experiment; that is, in which subjects behave according to theory only if their rivals decrease in number, not if they increase. Katzman (1995) also provides a theory indicating that the prevalence of demand reduction decreases with the number of participants even though some demand reduction will always be present. Engelbrecht-Wiggans et al. (2006) also establish that there is no difference in the first-unit-bid between the uniform auction and the Vickrey auction when the number of bidders increases from two to three (or five), even though the second-unit-bid is always greater in the Vickrey auction.

Concerning demand reduction, Noussair (1995) showed in a seminal paper that the bid on the second unit was always lower than its value, in contrast to the first bid, which was always demand-revealing. The degree of under-revelation depends on whether the bid sets the price or not. Ausubel and Crampton (2002) provide a formal proof of demand reduction in the uniform auction. Katzman (1995) and Engelbrecht-Wiggans and Kahn (1998) also analyze auctions that involve demand reduction.

Demand reduction has been confirmed in experimental research to various degrees, for example in a field experiment by List and Lucking-Reiley (2000) in which two players with two-unit demand bid for two units through the uniform price, the English, or the Vickrey auction, also replicated in a laboratory experiment by Porter and Vragov (2006). Another laboratory experiment where demand reduction is confirmed is by Kagel and Levin (2001); they let one bidder with two-unit demand compete against a robot bidder with unit demand and playing the dominant strategy.

With respect to learning, we have the evolutionary paradigm, or what Nelson and Winter (2002) call the "competence puzzle", which roughly means that individuals typically do not have the vast computational and cognitive powers that are imputed to them by the optimization-based theories (such as that in this article). But, since learning, guided by clear short-term feedback, can be remarkably powerful even in addressing complex challenges, the evolutionary response to the competence puzzle focuses on the role of learning and practice.

The research on the winner's curse is vast, starting with Capen et al. (1971) who claimed that oil companies suffered from low returns. A comprehensive survey of theory and experiments in single-unit, common value auctions is offered by Kagel and Levin (2002). They show that the WC is pervasive across various types of auctions and is not eliminated, only somewhat mitigated, by experience or even by using expert bidders. But experimental studies on common value, multi-unit auctions are scarce.

But one, notably, is Ausubel et al. (2009), which experimentally tests alternative auction designs suitable for pricing and removing troubled assets. They make use of the same static and dynamic uniform auction as this study and Engelmann and Grimm (2009) above, except that their dynamic format is an Ausubel descending clock auction. The units for sale are not identical, and they sell the units individually or as pooled units. And, for some sessions, bidders also know their liquidity needs. They find that the static and dynamic auctions resulted in similar prices. However, the dynamic auctions resulted in substantially higher bidders' payoffs, which enabled the bidders to better manage their liquidity needs. The dynamic auction was also better in terms of price discovery, as well as for reducing the bidder error.

Another study is Manelli et al. (2006) which experimentally compares the static Vickrey auction with the Ausubel auction, also known as the dynamic Vickrey auction, in both an IPV setting and an interdependent value (IV) setting, in which the values are affiliated. They conclude that due to overbidding in both types of auctions, but slightly more in the Vickrey auction, the revenue from the Vickrey auction is greater, while the efficiency is lower in the Ausubel auction. But in the IV setting, they observe less overbidding and a trade-off between efficiency and revenue; the Vickrey auction is more efficient while the revenue is higher in the Ausubel auction.

3 Experimental Design

The experiment used students from KTH (the Royal Institute of Technology) as experimental subjects. They were from different Master's programs in Engineering, and the experiment took place in May 2009. In total, 152 unique subjects participated in the experiment.

The subjects were recruited for computer sessions consisting of a series of auction periods. Each subject participated in one of three possible auction formats; hence, the design is between subjects. In each period, two identical units of a commodity were sold to the two highest bids, and these two bids could come from the same bidder or two different bidders. The units had no meaning for the participants apart from the money they could bring forth. Only the subject(s) who won the units earned profit(s), calculated as the induced value of the item minus the price paid for it. When there were ties, the winning bids were randomly selected.

The procedure for generating the common value was as follows: The value of the units for sale was generated by two random integers which were added together. Both units had the same value for the potential buyers, i.e. bidders had flat demand curves. To construct a tight market, we let the integers be chosen from the set $\{1, 2, \ldots, 6\}$. Thus, the possible values for each item were $\{2, 3, \ldots, 12\}$. The bidders were not fully informed about this value, though. Rather, each bidder was independently and randomly shown one of the two integers as a private signal. For expository reasons, we displayed these signals as dice. Subjects were told that two dice were rolled and added up as the common value, but each of them was only allowed to see one of the dice, independently of each other. Thus, the distribution of both the value and the signal was common knowledge. When the die was displayed on the screen, the subjects bid on both units. Only integer-value-bids between 0 and 12 were allowed. Subjects were also informed that the order of the bids was irrelevant. Bidder instructions are find in Appendix C.

Hence, a common value environment is created where private signals may be used to create unbiased estimates of the value of the items. If t_i is the signal, the common value will lie in $V = t_i + \{1, 2, \ldots, 6\}$ and an unbiased estimator of the value (ex ante) is then $\hat{v}_i = t_i + \frac{7}{2}$. Thus, the signals are positively correlated (affiliated) with the value. The underlying distribution of the private signals in the experiment was common knowledge; that is, everyone was told that she would see one of the two dice and her competitors might, but not necessarily, see the same die.

The group size was limited to two, three and four participants, respectively. Two approaches to allocating subjects to groups were used. In the first, each participant always competed against the same number of opponents, but not necessarily against the same opponents. Before each new round, all participants *within each group size* were re-randomized against each other. In the second approach, *all* participants were re-randomized against each other before each new round, irrespective of the group size. The reason for re-randomizing each new round was to counteract subject-specific effects and tacit collusion. Moreover, in advance of every new round, the common value of the last round was displayed on the screen alongside the two winning bids and the price paid for the two units. Moreover, to ensure that comparisons among auction formats were not driven by particular configurations of value, the two integers constituting the value were randomly generated for each new auction.

Each subject got SEK 100 as a participation fee, or show-up fee, and a starting balance of SEK 50 to cover losses. Profit and losses were added to this balance. If a participant's balance went negative, he or she was suspended from the auction and had to leave (with the participation fee). The others were paid in cash at the end of the experiment.

One of the justifications for the starting balance is that, even if participants play the risk-neutral Nash equilibrium, losses may occur. A starting balance also imposes opportunity costs for overly aggressive bidding, and is enough for errors made during bidding and a reasonable return for participating if aggressive bidders shut them out of the auction.

4 Theory and Hypotheses

The theoretical model in this article originates from Ahlberg (2009) where it is presented more thoroughly. An excerpt from it is available in Appendix A.

Starting with the revenue question, and beginning with 2-player groups, equation 9 in the Appendix shows there to be a unique equilibrium strategy in the discriminatory auction; it prescribes the player to bid equal amounts on both units. For the uniform and Vickrey auctions, there is no unique strategy; the uniform auction has a multitude of equilibria, whereas the Vickrey auction has dualistic equilibria. One thing in common for both, however, is that they have extreme demand reduction equilibria, i.e. equilibria that prescribe a zero bid on the second unit and thereby transform the players into single-unit demanders. Equations 11 and 13 in the Appendix also show these to be the payoff-dominating equilibria. The equations show that the equilibrium bid for the first unit is to bid the conditional expected value, which makes the equilibrium risk dominate the other equilibria. (Bidders could also bid above this value for the first unit, but with a higher risk.)

Thus, using the unique strategy in the discriminatory auction and the payoff-dominated equilibria in the two other auctions, the following expected revenues for a two-player game emerges (see eq. 14 and 15):

$$\begin{aligned} E[R^{D}(2 \text{ Players})] &= 11.22 \\ E[R^{U,V}(2 \text{ Players})] &= 0 \end{aligned}$$

where U stands for the uniform, D for the discriminatory and V for the Vickrey pricing rule.

Since the ex ante expected value for the two units for sale is 14, we see that, in the discriminatory auction, the seller captures the major part of the value at stake, but zero in the two other formats. Thus, the discriminatory auction gets the highest ranking. As regards the two other formats, the payoff-dominating equilibrium for the uniform auction is somewhat more robust (in the uniqueness of the bidder's best response) than the payoff-dominating equilibrium of the Vickrey auction (see section 9.5.3), which thus indicates that the Vickrey auction should deliver weakly more revenue than the uniform auction.

When there are more than two players in the auctions, the extreme demand-reduction strategy of bidding zero on the second unit disappears. (Since it is always an equilibrium to bid the conditional expected value on the first unit, notwithstanding the group-size, the price-setting bid will never be zero; thus, a zero-bid on the second unit does not gain anything.) However, the bidders must now be cautious about the first unit bid, as it can be the price-setting bid. Following the comparison of conjectures 7 and 8 in Appendix B for the uniform auction with the strategies for the discriminatory auction in section 9.5.1, we have that the uniform auction always gives a greater revenue than the discriminatory auction when there are 3 or 4 players. Hence,

$$
\begin{aligned}
&E[R^U(3\text{ Players})] > E[R^D(3\text{ Players})]\\
&E[R^U(4\text{ Players})] > E[R^D(4\text{ Players})].
\end{aligned}
$$

In the Vickrey auction, all pure equilibria disappear due to non-core outcomes since there is always a coalition for which the total payoff becomes higher; notwithstanding the individual strategy. Thus, we can formulate the following hypothesis:

Hypothesis 1 (Revenue Comparison) *If there are only two bidders in the auction, the discriminatory auction will outperform the two other mechanisms, and the Vickrey auction will have a marginally higher rank than the uniform auction. With more bidders, the uniform auction is likely to give more revenue than the discriminatory auction.*

The second topic is how the number of bidders influences the bidding and hence, the revenue. The expected revenue is calculated using the unique strategies for the three group sizes of the discriminatory auction (see eq. 14 in the Appendix):

$$E[R^D(2\text{ Players})] = 11.22 \tag{1}$$
$$E[R^D(3\text{ Players})] = 12.38 \tag{2}$$
$$E[R^D(4\text{ Players})] = 12.63. \tag{3}$$

If we start with a two-player game and increase the number of bidders by one, the revenue increases by almost 10 percent. If we go from the three to the four-player game, the revenue increase is just 2 percent. Since going to five players only increases the revenue by 1.5 percent, we also see that using four players actually captures the idea of "many" bidders. Thus, in this setting, the expected revenue seems to increase concavely with the number of bidders.

The same is also true for the uniform auction, because of diminishing incentives for demand reduction the more players there are. From conjectures 7 and 8, we have that both the bids on the first and the second units (weakly) increase with the number of participants. Thus, in terms of expected revenue, we have

$$E[R^U(2 \text{ Players})] > E[R^U(3 \text{ Players})] > E[R^U(4 \text{ Players})].$$

Going from two to three players, we have a clear revenue ranking following the disappearance of the extreme demand reduction. The next step is less pronounced, but at least there is a distinct difference in revenue.

Thereby, we have the following hypothesis:

Hypothesis 2 (The number of bidders) *The revenue will increase with the number of bidders. Moreover, we expect to see a greater difference between two and three-player groups as compared to three and four-player groups. This should be true for both the discriminatory and the uniform auctions.*

A third hypothesis concerns the uniform and the Vickrey auctions. We have seen (above) that, when there are only two bidders, there could be zero bids on the second unit. This, in turn, gives zero revenue to the seller. Hence,

Hypothesis 3 (Demand reduction) *When there are two bidders, bids on the second unit will be (much) lower than the expected value (i.e. under-revealing) in the uniform and Vickrey auctions.*

The fourth hypothesis concerns profit maximization and learning, i.e., evolutionary aspects of the bidding process. Each settled round gives feedback on the performance which, correctly interpreted, gives an indication of how to bid in the next round. So even if subjects do not understand how to compute an equilibrium strategy, they may roughly learn a rule of thumb.

Thus, an iterative process may be needed to approach equilibrium play.

Hypothesis 4 (Learning) *Subjects are likely to use strategies closer to (theoretical) equilibrium play over time.*

Last, the winner's curse (WC) is scrutinized. The first to recognize the WC was Capen et al. (1971) who argued that the low rates of return among oil companies in the 1960s and 1970s on OCS lease sales, year after year, resulted from bidders' ignorance about the informational consequences of winning.

Hence, we define the WC as the adverse selection effect of bidders neglecting the information a win will produce. That is, that the announcement of winning leads to a decrease in the estimated value of the objects, if not accounted for when bidding (given a symmetric game and that the high signal holder(s) win(s) the objects). The underlying cause in this study is that, even though the signal plus the expected value (EV) of the other integer is an unbiased estimator of the value, the max operator of all $t_i + 7/2$ is not; it is a convex function and thus overestimates the value.

In the present design, the lower bound estimate of the value is $(t_i + 1)$ and the upper bound is $(t_i + 6)$. The strategy of bidding the risk-free lower bound strategy never yields a negative payoff, whereas bidding above the upper bound would ensure a negative payoff. [2] The unbiased, though naive, EV of the items is $E(v|t_i) = (t_i + 7/2)$. It is naive in the sense that it is the EV, independent of winning the item(s). Define $E(v|t_i > t_{-i})$ as the EV, for player i, conditional on having the highest signal, t_i. Since the informational content of winning leads to a decrease in the estimated value, and cannot be lower than the lower bound, it must lie in the interval $\{t_i + 1, t_i + 7/2\}$. For $t_i \in \{1, 2, \ldots, 6\}$ it is $E(v|t_i > t_{-i}) = 3t_i/2$.

A bidder who does not take this fully into account and uses the naive EV instead of the appropriate EV conditional of winning when placing her bids could, upon winning, pay more than the estimated value of the object(s). The systematic failure to account for this is referred to as the winner's curse.

The difference $E(v|t_i) - E(v|t_i > t_{-i})$ decreases with the signal t_i; meaning that the greater the signal, the less the bidder has to shade the bid to account for the winner's curse. Or, stated differently, it is worse to find out that one won with a low signal rather than a high.

In the present analysis, we discriminate between bidding in the WC interval, which s then above $E(v|t_i > t_{-i})$ up to the naive $E(v|t_i)$, and bidding above the latter. And since it is risk-free to bid $(t_i + 1)$, the interval in question, i.e. the WC interval, becomes $\{t_i + 2, t_i + 3\}$. [3] The reason for the division of the intervals is that, theoretically, bidding above the naive EV has nothing to do with the WC. Bidding above the naive EV will, on average, produce a negative profit. But since this discrimination is not made in other experiments, we will also pool the result.

[2] That is to say, bidding in the discriminatory auction. For the uniform and Vickrey auctions, where players do not pay what they bid, the word bidding should be interpreted as paying.

[3] Bidding above the unbiased estimate $t_i + 7/2$ is not rational, ex ante. But, there is also what Holt and Sherman (1994) call a loser's curse. The expected value of the item, conditional on not winning, is greater than the naive expected value. In this model it is, for bidder i, $E(v|t_i < t_{-i}) = \frac{3t_i+7}{2} > t_i + 7/2 = E(v|t_i)$. The difference $E(v|t_i < t_{-i}) - E(v|t_i)$ increases with the signal t_i, meaning that the greater the signal, the more the bidder might have bid to account for the loser's curse. By inspection of the data, we conclude that in this experiment, the loser's curse is non-existent.

Moreover, our way of generating the signal from the CV makes the CV upwardly biased from the signal. That is to say, even though $\hat{v}_i = t_i + \frac{7}{2}$ is unbiased, t_i underestimates the CV. In, for example, Kagel et al. (1987), where the signal is drawn from a set consisting of $\pm\epsilon$ of the CV, there is a certain region where the signal by itself is an unbiased estimator of the CV. We believe that the latter method will produce more WC due to the fact that in fifty percent of the draws, the signal is below the CV.

There is vast experimental evidence of the WC for both inexperienced players and professionals in single-unit auctions, see Kagel (1995). Due to the inherent demand reduction equilibria in two of the auction formats, the WC should be lower on the second unit for sale. Thus, we conclude that:

Hypothesis 5 (Winner's curse) *The winner's curse will be apparent, but not so much as reported in other experiments since the common value in the present experiment is biased upwards. And the WC should be considerably smaller for the second unit, as compared to the first.*

5 Experimental Results

We conducted 15 or 20 rounds of bidding for each subject; the number was stochastically determined, not known in advance by the subjects (they did not know how many rounds they were going to play). The data description is in Table 1, which shows, for each format, the number of subjects, how many rounds there were, the number of unique observations, and the average profit. Each format consisted of groups of two, three and four bidders.

	No. of subjects	No. of rounds	Unique observations	$\bar{\pi}$
Discriminatory	62	345	941	0.37
Uniform	44	256	745	0.36
Vickrey	46	286	777	1.04

Table 1
Data summary

There were two experimental designs; one configuration where subjects remained in the same group size in all rounds, i.e. the number of competitors was always constant for them, but the competitors changed; and another where subjects were randomized without any constraints in all rounds, i.e. both the number of individual competitors and the competitors changed. But, when using the highest (or lowest) bid as the dependent variable in an ordinary least squares regression, these different designs do not have any significant influence. Nor when the design interacts with rounds, auction format or group size is there a significant effect on the highest (or lowest) bid. Therefore, the data from the two experimental designs is pooled.

We use a *first unit bid* and a *high bid* interchangeably, meaning the (weakly) highest bid of the two bids that each subject submits. Likewise, a *second unit bid* and a *low bid* refer to the (weakly) lower bid of the two. The non-parametric Wilcoxon(-Mann-Whitney) rank sum test is used for comparing data between treatments, if not stated differently. There seems to be no problem with dependencies within subjects, nor within groups. We have made tests with OLS and Panel data (random effects) models with revenue (price) as the dependent variable. Revenue is explained by format, group-size, round and design. We have also made interactions between group-size and format on the above. Moreover, we have used both the difference in bids and equilibrium bids as dependent variables, explained by the same covariates as the former, and interactions between group-size and format. The below presented results only changed marginally and, thus, the conclusions still hold. (The OLS regression on revenue can be found in Appendix B.)

Last, in the discriminatory auction, roughly 11 percent of the subjects went bankrupt. For the uniform auction, that portion was only about 4 percent, whereas the Vickrey auction had zero bankruptcies.

Table 2 displays the average revenue for different auction formats (rows) and group sizes (columns). The numbers inside the brackets are the revenues in Bayesian equilibrium, to the extent that it is found. (See eq 1 for the discriminatory auction. The uniform and Vickrey auctions are the payoff dominating Bayesian equilibrium, that is, the extreme demand reduction equilibrium.)

Group Size	2	3	4	pooled
Discriminatory	11.70 (11.22)	13.41 (12.38)	14.12 (12.63)	12.81 (11.90)
Uniform	11.70 (0)	12.89	14.69	12.97
Vickrey	9.49 (0)	11.65	14.59	11.22

Table 2
Average revenue, with predicted revenue inside the parenthesis.

Hypothesis 1:

Overall, there are no significant differences between the discriminatory and the uniform auction as concerns concerning revenue. The Vickrey format is inferior, especially for small group sizes. Interestingly enough, the larger the group size, the closer the revenues are between the auctions. Looking at 4-groups alone, the formats are not significantly different from each other. In the other group-sizes, and when groups are pooled, the Vickery auction collects significantly less revenue than both other formats ($p-$values < 0.01).

Result 1 (Revenue Comparison) *In 2−player groups, the discriminatory together with the uniform auction collects significantly more revenue than the Vickrey auction. This is contrary to the hypothesis that the discriminatory auction should outperform the uniform auction. When there were more bidders, the hypothesis was that the uniform auction should give a weakly higher revenue than the discriminatory auction; which did not happen either.*

Hypothesis 2:

Table 2 also indicates that larger auction groups give more revenue. All p-values except one are below 0.01; the one above is 0.1156 and concerns the discriminatory auction between 3- and 4-groups.

Result 2 (The number of bidders) *The result for both the uniform and the Vickrey auctions supports the hypothesis that the revenue increases with the number of bidders. For the discriminatory auction, the result is not as strong because of the high significance level (*15*-percent level) between two group sizes; but the result, therefore, verifies that the revenue increase is concave in that format.*

Hypothesis 3:

Demand reduction, or bid shading, means that bidders do not apply the demand-revealing strategies. In the discriminatory auction, the unique symmetric strategy for bidders is to bid equally on both units. Hence, there should not be any demand reduction in that format. However, in both the uniform and the Vickrey auctions, there are optimal strategies that are both demand revealing and not. In the latter strategies, the bidders will always bid below the expected value for the second unit, possibly zero. We will also report demand reduction for the discriminatory auction.

For all three formats, first, Table 3 shows the frequency of the bid-spread and, second, the value of the bid-spread, third, given that the bids are not equal, what is the frequency for Bid 1 to be above the EV and, finally, given that the bids are not equal, with what frequency Bid 2 is below the EV. In other words, the next to last column shows if the subjects engaged in a bid-spread overbid or not for the first unit, and, in the last column, if they underbid or not for the second unit.

	Bid-spread	Bid-spread value	Bid 1 > EV	Bid 2 < EV
Discriminatory	0.58	0.88	0.15	0.90
Uniform	0.78	1.85	0.55	0.70
Vickrey	0.73	1.41	0.50	0.69

Table 3
Frequency and value of bid-spread.

The uniform and the Vickrey auction have overlapping strategies, given the same signal, even though the price rule differs between them. The discriminatory auction has both different strategies, given the same signal, and a different pricing rule. This explains the similar frequencies in the last two formats as well as the great discrepancy between them and those of the discriminatory auction.

Hypothesis 3 suggests that there should be, if not complete, at least a great under-revelation for the second bid in both uniform and Vickrey auctions. First, not seen in the table, we conclude that there is very little extreme demand reduction behavior, that is, zero bids on the second unit, even though this is the payoff-dominating strategy in games with two players for the uniform and Vickrey auctions (only 5 percent of the bids in the uniform auction, and 3 percent in the two other formats). There is no significant difference in zero-bids between the formats.

If we use the bid-spread as a measure for demand reduction, we see that the uniform and the Vickrey auction have quite the same frequency of demand reduction; whereas the discriminatory auction has a lower frequency. Nevertheless, all three formats are significantly different from each other (p-values < 0.022). Comparing the values, there is a larger spread between the formats; which is a reflection of the significant difference between them (p-values < 0.001).

The next-to-last column tells us that, for the uniform and Vickrey auction, half of the subjects who engage in demand reduction also bid above the value on the first unit bid. Thus, we must look at the second unit bid to understand if demand reduction is present. (It could be the case that both bids are above value, which would then not really be demand reduction.) This cannot be compared to the much lower frequency in the discriminatory auction, where the winning bids become the price. In the last column, we see the frequency of under-revealing bids. All bids should be under revealing in the discriminatory auction, which is almost the case. More interesting, both the uniform and the Vickery auction have about seventy percent second unit bids below the expected value.[4] The two formats do not differ significantly from each other in this respect.

Result 3 (Demand reduction) *We find evidence of demand reduction on the second unit but, in contrast to hypothesis* 3*, very few zero bids on the second*

[4] The 70% share of bid 2 below EV almost coincides with the Porter and Vragov (2006) result. They had a 68% share in an IPV experiment with two bidders and two units for sale. But they just got a 30% share for the Vickrey auction. (One must be cautious when making a comparison with their results since they have a different value system to the one in this study.) Moreover, we are now using the EV, and not the EV conditional of winning, since we are looking at all subjects.

unit. All formats differ significantly from each other on both the frequency and the value of the bid-spread. And since the uniform and Vickrey auctions do not differ in the under revealing of the second unit bid frequency, we conclude that the uniform auction produces more demand reduction than the Vickrey auction. As for the discriminatory auction, where the symmetric equilibrium does not predict demand reduction, the format has significantly lower values and frequency compared with the other two.

Hypothesis 4:

For the discriminatory auction, we measure learning as the share of first and second unit bids, consistent with the theoretical, extended-equilibrium strategy where the extended-equilibrium strategy is defined as: $b_1, b_2 \in (b^* - 1, b^* + 1)$. Does the share of bids in this interval increase with the number of rounds played?

Equipped with this definition, a learning effect in the discriminatory auction can be seen in Table 4. Moreover, this effect is concave. Between rounds $4-6$ and the middle rounds, it is significant at the one-percent level, and between the middle rounds and the last three rounds, it is significant at the five-percent level.

Round	$4-6$	$11-13$	$18-20$
Discriminatory	61 %	75 %	85 %

Table 4
Frequencies of (extended) optimal bids

Regarding the uniform and the Vickrey auctions, we do not have any equilibrium strategy prediction for more than two players. Thus, we concentrate on 2-player groups, and measure learning as finding the payoff-dominating equilibrium strategy. Hence, do subjects increase the number of zero-bids on the second unit along with the rounds played, or, at least lower the second unit bid as the session continues?

There was no such effect in the Vickrey auction; whereas there was a tendency to it in the uniform auction. That is to say, the p-value was 0.1125 when comparing the second unit of the early rounds with the last three rounds.

Result 4 (Learning) *In the discriminatory auction, the subjects moved towards the optimal strategy over time, consistent with hypothesis* 4. *The subjects also continued to learn in later periods, but to a lesser extent. That is, the learning effect is concave (at least between the measuring points). In the uniform auction, the learning was barely significant, but the subjects seemed to weakly understand the demand-reduction equilibria over time (rounds).*

Comment:

There is especially one odd result here when compared to theory, which spurs both anomalies in hypotheses 1 and 3. The subjects' bids in 2-player groups were expected to be (much) lower in the uniform and the Vickrey auctions. Even if the revenue does not go to zero as predicted by theory, it should at least be much lower (than the discriminatory auction). Maybe the competitive element, or the joy of winning, [5] overtook any rationale in these groups. Even though some subjects understood having to play zero on the second unit and high on the first, their opponents seldom did. Another prominent feature in the experiment is the low revenue outcome in the Vickrey auction for two- and three-player games. Supposedly because of its complicated nature, the subjects did not seem to understand this.

Hypothesis 5:

As described above, the WC interval is defined as bids above the EV conditional on winning, $EV_c = E(v|t_i > t_{-i}) = 3t_i/2$, up to the naive EV, $EV_n = E(v|t_i) = (t_i + 7/2)$. Since it may not be intuitive to grasp the underlying cause of the winner's curse, i.e. the convexity of the max function, bids in this interval could be rationalized on the basis of the fact that they are (at least) below the naive expected value. However, bids above $E(v|t_i)$ are, on average, never individual-rational since they produce a negative profit on average. [6] Moreover, to separate the random component from the actual bid, we distinguish between bidding in the WC interval and actually experiencing a negative profit, i.e. suffering from the winner's curse, in Tables 4 to 6 below.

In the uniform and Vickrey auctions, the bid is just a proxy for the price, since subjects do (often) not pay what they bid. The price-setting bid could come from any bidder in the uniform auction but, in the Vickrey auction, it is always another player's bid that becomes the price-setting bid. Thus, in these two formats, we are measuring more like a generalized WC; a WC within each group. All bid frequencies, or prices, in table 5 are conditional on both winning and having the high signal. That is, as stated in the above paragraph, even though we are using bid 1 and bid 2 in the table, it is the price that these bids generated that we are measuring for the last two formats.

The table is to be interpreted as follows: In the discriminatory auction, 35 percent of the second-unit bids were in the WC interval. Of these, 57 percent de facto gave a negative profit. Thus, a total of 20 percent second-unit bids gave negative profits.

[5] Cox et al. (1992) tried to explain overbidding in IPV first price auctions with 'joy of winning' and Cooper and Hanming (2008) partly support a modified version of the 'joy of winning' hypothesis in an experimental study of the IPV second-price auction.
[6] It could be individual-rational for 2-player groups in the uniform and Vickrey auctions. But that hinges on how the other player bids, and it is still quite risky.

	$EV_n \geq b_i > EV_c$		Winner's curse		Total	
	b_1	b_2	b_1	b_2	b_1	b_2
Discriminatory	0.42	0.35	0.42	0.57	0.18	0.20
Uniform	0.37	0.32	0.47	0.42	0.17	0.13
Vickrey	0.36	0.30	0.34	0.33	0.12	0.10

Table 5
The frequency of winner's curse bids and actually experienced winner's curse.

At a first glance, there is no significant difference between the first and second unit prices within each auction type (the p-values starting from 0.1229 and rising.). But looking more closely reveals a pattern due to the lower frequency of second unit bids/prices vs. first unit bid/prices in all three formats. (There is an interval ranging from 0.05 to 0.07 between the two bids.) This is also confirmed with a p-value of 0.0545, when testing all formats together for differences between the two bids. Hence, we have a distinct difference between bids for subjects when pooling all auction formats.

Switching to a comparison between auction formats, and once more testing for differences between bids in the WC-interval, we only find a difference between the discriminatory and Vickrey auctions; the p-value is 0.0928 when comparing first unit prices.

Since the difference between the first and second unit bids is weak in this case, Table 8 shows the results when pooling first and second unit bids. It can

	$EV_n \geq b_i > EV_c$	Winner's curse	Total
	b_1 & b_2	b_1 & b_2	b_1 & b_2
Discriminatory	0.40	0.46	0.18
Uniform	0.36	0.46	0.17
Vickrey	0.34	0.34	0.12

Table 6
The frequency of winner's curse bids and experienced winner's curse, when bids 1 & 2 are pooled.

be seen that the uniform and the discriminatory auctions are almost identical when analyzing the total WC. The Vickrey auction has almost the same frequency of bids in the WC interval, but experienced WC is lower; hence, it has a roughly 30 percent lower total WC when compared with the other two. As above, the only difference in bids between the auctions is between the discriminatory and Vickrey auctions; now the p-value becomes somewhat lower, namely 0.0692, when the first and second unit prices are pooled. Still, the significance levels are quite weak.

When bids/prices above the expected value are scrutinized which, as mentioned above, is not really a WC problem but shown here for reference, the following table emerges (Table 7). Only the bid for the second unit in the discriminatory auction differs significantly from the others when analyzing bids above EV_n. Thus, the pooled results (b_1&b_2) are also shown in the table, as is the total WC. (To be comparable with the analysis above, all bid frequencies, or prices, in the table are conditional on both winning and having the high signal.)

	$b_i > EV_n$			Negative profit			Total
	b_1	b_2	b_1&b_2	b_1	b_2	b_1&b_2	b_1&b_2
Discriminatory	0.12	0.08	0.11	0.80	0.77	0.79	0.09
Uniform	0.13	0.19	0.14	0.71	0.64	0.69	0.10
Vickrey	0.11	0.11	0.11	0.67	0.89	0.72	0.08

Table 7
Frequency of bids above the (naive) expected value, and the negative payoff.

As for the discriminatory auction, subjects are more cautious when bidding on the second unit, and the lion's share of the second unit bids give a negative payoff, but the analysis of the second unit bids is to be taken with caution due to lack of data. The lack of data on the second-unit bids is shown in both columns of pooled bids, since the pooled results highly resemble the first unit-bids. But, in total, the formats are not significantly different from each other in both bidding above expected value and making a negative profit.

The result of pooling all bids above EV_c is shown in Table 8. The formats are not significantly different from each other, so the ranking is ambiguous.

	Bids giving negative profit
	b_1 & b_2
Discriminatory	0.28
Uniform	0.31
Vickrey	0.23

Table 8
The percentage of bids giving negative profits, in total

Result 5 (Winner's curse) *The winner's curse is highly visible, but does not have as large an effect on outcomes as in results from earlier experiments, c.f. Kagel and Levin (2002), for inexperienced bidders in single unit auctions. The cases of WC are also robust across the sample population and not just for a couple of bidders. Across group sizes, there is no difference between the sizes in bidding in the WC interval.*

Comment:

As stated at the beginning, for players in 2-player groups, it could be an equilibrium strategy to bid on or above any of the expected values. But this is only the case if both bidders bid zero on the second unit, which never happened. Thus, bids from 2-player groups are included in the above results.

The impact-differences of the winner's curse across distinct set-ups, i.e. other experiments, could be explained from the construction of the common value interval and the private signal generation. Here, if a player got a 6 (1) as a signal, he/she knew that the signal was the highest (lowest) possible. This is, of course, valuable information. One purpose of this set-up was to make the idea of common value clear and uncomplicated to understand and, thus, the winner's curse would be mitigated. This was the case in the present experiment. Moreover, due to demand reduction, the WC would be lower in multi-unit settings than in single-unit settings.

6 Discussion

First, we notice that as the number of players increases, the pricing rules converge in collecting revenue. When there were only two bidders in the auction, all formats were significantly different in revenue raising, but when there were four bidders, the difference became insignificant. Thus, attracting bidders, or ensuring competition, could be much more important than selecting the auction form.

There was one particularly odd result in the experiment, namely the high revenue for 2-player groups in the uniform auction. This was rather unexpected because of the anticipated low revenue equilibria outcome of this group. One possible explanation is the competitive element; subjects did not play the theoretical equilibrium at all; but wanted to win the object(s), no matter what the costs. Holt and Sherman (1994) explain this as the joy of winning phenomenon in their study. In the present study, it was encountered not only in this particular group size, but was pretty common in all group sizes in all auction formats.

As for the winner's curse, we chose to solely isolate the WC interval. Many experiments do not distinguish between the intervals and, thus, treat all bids above the EV conditional on winning as potential winner's curse bids, which, per definition, they are not. But, of course, all bids above the EV conditional of winning are dangerous and could give rise to a negative profit. Thus, we present the results from the bids above the naive EV, and then the experiment is comparable with the results from other experiments. Another issue concerning the WC was that it entailed no learning effects; subjects continued to suffer from the winner's curse in later rounds, and not just in the early rounds. They never really grasped the idea.

Contrary to the non-learning in the WC problem, there was another type of learning that we chose to discuss here because of lack of evidence in the data. Subjects learned in the course of play, i.e. they adapted to what the other player(s) did in the auction and bid according to that. In other words, they were trying to find a best-response function. Nonetheless, because of the common value structure, where the random component played a part of the profit earned, it is hard to see the evidence in the data.

All subjects were inexperienced players, and one must be careful in drawing policy recommendations from the result. But other research, Kagel and Levin (2002) for example, has shown that overbidding is a robust feature, not only for bidders with no experience, but also for professionals.

7 Conclusion

The present paper has studied the two most common auction formats used in the field, the discriminatory and the uniform auctions, as well as the Vickrey auction, a more theoretical format. All three formats make use of two treatments; first, varying the number of bidders and, second, repeating the auction several times inside each session.

The main conclusion was that the auction format is less important for revenue generation when the number of bidders is large; there were no significant differences in the revenue of the three formats when there were four competitors in the auction. Neither of the discriminatory and the uniform auctions could be distinguished as better at revenue generation than the other; only the Vickrey auction could be classified as inferior, compared to the others, when there were few bidders. One possible explanation for this could be the complicated nature of the Vickrey auction, which subjects had difficulties in understanding.

Interestingly enough in the experiment, almost no one understood the extreme demand-reduction equilibrium. Very few, indeed, grasped the idea of bidding zero when there were just two bidders in the uniform and the Vickrey auctions. Moreover, in the discriminatory auction, subjects learned to play equilibrium strategy over time.

The WC is still a problem; overall, subjects did not seem to understand the adverse selection effect that winning produces. Regardless of group size or auction type, the WC was always there for about 17 percent for the two most common auctions, and around 12 percent for the Vickrey auction. The WC in this study is defined as bids/prices between the expected value conditional on winning and the (usual, naive) expected value, not bids above this expected value. Bids above the naive expected value were somewhat less common, and quite the same in all three formats, around 9 percent.

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9 Appendix A

The theoretical model is found in Ahlberg (2009), but an excerpt is presented here.

9.1 The Model

In a Bayesian game, players update their prior beliefs by Bayes' rule, as well as the opponents' payoff functions, once they learn their types.

9.2 Posterior beliefs

Let the type-vector of all players except player i be denoted by $t_{-i} = (t_i, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n)$. Then, given a player's own type t_i, denote $\mu(t_{-i}|t_i)$ as player i's conditional probability, or posterior beliefs, about her opponents' types.

$$\mu(t_{-i}|t_i) = \begin{cases} \frac{1}{2^{n-1}} \cdot 1 + \frac{2^{n-1}-1}{2^{n-1}} \cdot \frac{1}{6} & \text{all types in } t_{-i} \text{ are equal to } t_i, \\ \binom{n-1}{x} \cdot \frac{1}{2^{n-1}} \cdot \frac{1}{6} & \text{At least one type in } t_{-1} \text{ is different from } t_i. \\ \frac{1}{2^{n-1}} \cdot \frac{1}{6} & \text{all types in } t_{-i} \text{ are different from } t_i. \end{cases}$$

where x is defined as the number of players who are of the same type as player i. The first equation works as follows: If all other players, except player i, see the same value as t_i, two things can happen. Either they all see the same die as player i, which happens with probability $\frac{1}{2^{n-1}}$, or at least one of the others sees a different die with the same value as t_i, which happens with probability $\frac{2^{n-1}-1}{2^{n-1}} \cdot \frac{1}{6}$. (The first term is the probability that at least one sees a different die and the second term is the probability that the die has the same value as t_i.)

The second equation says that if one of the non-i players sees a different value than player i, the belief for player i becomes $\binom{n-1}{x} \cdot \frac{1}{2^{n-1}} \cdot \frac{1}{6}$. In the last row, all non-i players see a different value than player i, and, since we only have two dice, they all see the same die. This happens with probability $\frac{1}{2^{n-1}} \cdot \frac{1}{6}$.

9.3 Strategy and payoff

The strategy for each player is a to assign two (integer-)bids, one for each item, from her signal. Formally, the strategy for player i is a mapping from her signal space $T_i = \{1, 2, \ldots, 6\}$ to the two-dimensional space of integers, $b_i : T_i \to \mathbb{Z}^2_+$, where $b_i(t_i) = (b_{i,1}, b_{i,2})$.

Let D_1, D_2 be the random variables describing the outcome of the two integers, or dice, respectively. Then, the value for each bidder is the realization of the two variables, hence $v = d_1 + d_2$. Define $k_i(b)$ as the number of items won by player i if strategy profile b is employed.

Now, let $b = (b_1, \ldots, b_n)$ be a strategy profile and let $v = d_1 + d_2$ be a random variable. Then, player i's payoff function $\pi_i^l : b \to \mathbb{R}$ is defined as

$$\pi_i^l(b, v) = k_i(b)v - p_i(b). \tag{4}$$

Thus, the payoff is the number of items won multiplied by the realized value of these items, minus the price the winner has to pay for them.

9.4 *Expected value functions*

Since players only get to see one integer, i. e. their signal, they have to use the expected value when calculating their value, which is $v(t_i) = t_i + 7/2$, that is, the value of her signal plus the expected value of the other die. But in a Bayesian game they also need to calculate their competitors' value, given their own signal. This *conditional expected value* for the other players is dependent on how many players there are in the game/auction.

The fact that induces this is that they can all see the same integer or different integers. The more signals (players), the more accurate becomes the conditional expected value. That is, with many bidders, we approach the true value. This is an application of information aggregation, studied by Wilson (1977). The conditional expected value is defined as:

$$\begin{aligned} v_i(t_{-i}|t_i) &= v(t_{-i}|t_i) \\ &= \begin{cases} \frac{1}{2^{n-1}}(t_i + \bar{t}) + \frac{2^{n-1}-1}{2^{n-1}} \cdot 2t_i & \text{all } t_j = t_i, \\ t_i + t_j \qquad \text{(where } t_j \in t_{-i}) & \text{some } t_j \neq t_i, \end{cases} & (5) \\ &= \begin{cases} \frac{2^n-1}{2^{n-1}} t_i + \frac{1}{2^{n-1}}\bar{t} & \text{all } t_j = t_i, \\ t_i + t_j \qquad \text{(where } t_j \in t_{-i}) & \text{some } t_j \neq t_i, \end{cases} & (6) \end{aligned}$$

The first row in equation (5) says that if the non-i players are of the same type, t_i, as player i, two things can happen. Either they see the same die as player i, which occurs with probability $\frac{1}{2^{n-1}}$, or at least one of them sees a different die. The value for the former becomes $t_i + 7/2$ for player i, while the value for the latter becomes $t_i + t_i = 2t_i$.

In the second row of the same equation, we see the value if one, or both, is of a different type than player i. Then, since there are only two distinct integers, the value becomes the sum of the integer values. Equation (6) is just a simplification.

The term expectation above means expectation over the possible outcomes of the integer values. From player i's perspective, if we also take expectations over all t_{-i}, we get the expected value for a competitor, given player i's type. That is, we must combine the posterior beliefs with the conditional expected value to get the expected value for any competitor. Hence, the expected value for a competitor to player i is defined as

$$E_{t_{-i}}[v(t_{-i}|t_i)] = \sum_{t_{-i}} \mu(t_{-i}|t_i)v(t_{-i}|t_i). \tag{7}$$

If we also take the expectation over all t_i, the terms will sum up to seven as they should. But for any given t_i, the expected value for a competitor will not be $t_i + 7/2$. This is the case since we have to take into account that the competitor may get the signal from the same die as player i.

9.5 Equilibrium

A *Bayesian equilibrium* of this game with a finite number of types t_i, for each player i, and a common prior distribution μ, and pure strategy spaces T_i is a Nash equilibrium of the "expanded game" where each player i's space of pure strategies is the set $(\mathbb{Z}^2_+)^{T_i}$ of maps from T_i to $\mathbb{Z}^2_+$.

Given strategy profile $b(\cdot)$, and $b'_i(\cdot) \in (\mathbb{Z}^2_+)^{T_i}$, let $(b'_i(\cdot), b_{-i}(\cdot))$ denote the profile where player i plays $b'_i(\cdot)$ and the other players follow $b(\cdot)$, and let

$$(b'_i(t_i), b_{-i}(t_{-i})) = (b_1(t_1), \dots, b_{i-1}(t_{i-1}), b'_i(t_i), b_{i+1}(t_{i+1}), \dots, b_n(t_n))$$

denote the value of this profile at (t_i, t_{-i}). Then, since all types have positive probabilities, the bid/strategy $b_i(t_i)$ is a (pure strategy) Bayesian equilibrium if player i maximizes her expected utility conditional on t_i for each t_{-i}:

$$b_i(t_i) \in \arg\max_{b'_i \in \mathbb{Z}^2_+} \sum_{t_{-i}} \mu(t_{-i}|t_i)[k_i(b'_i, b_{-i})v(t_{-i}|t_i) - p^l_i(b'_i, b_{-i})]. \tag{8}$$

We only allow integer-value-bids[7]. Since the value function is symmetrical and we have a symmetrical joint distribution, only types will be of importance when bidding; thus, we look for a symmetrical equilibrium.

9.5.1 The Discriminatory auction

In this auction, conditional on winning, for each item won, every bidder pays the price of her bid on that item.

[7] A pure strategy Bayesian equilibrium in $\mathbb{R}$ does not exist.

From equation 8 in the Appendix A, we can derive the following unique pure Bayesian equilibrium strategy for two bidders:

$$b^*(t_i) = (t_i + \lceil \frac{t_i}{3} \rceil, t_i + \lceil \frac{t_i}{3} \rceil), \tag{9}$$

where $\lceil x \rceil$ is a ceiling function which maps x to the smallest following integer, i.e. $\lceil x \rceil = min\{n \in \mathbb{Z} | n \geq x\}$. The striking feature is that players bid the same amount on both units. [8]

When we increase the bidders by one, all bidders but the type-6 player bid the same as in a two-player game. The type-6 players raise their bids on both units by one increment unit.

If we look at the four-player game, we get the same increase in the bids for the type-6 players as in the three-player game, but a reduction in the bids for the type-1 players. The reduction is one increment. The optimal strategy when there are four players can then be written as:

$$b^*(t_i) = (t_i + \lfloor \frac{t_i}{2} \rfloor, t_i + \lfloor \frac{t_i}{2} \rfloor), \tag{10}$$

where $\lfloor x \rfloor$ is a floor function which maps x to the largest previous integer, i.e. $\lfloor x \rfloor = max\{m \in \mathbb{Z} | m \leq x\}$.

9.5.2 Uniform auction

Conjecture 6 (Two players) *In a two-player game, no other equilibrium payoff dominates the following:*

$$b^*(t_i) = (b_1^*, b_2^*) = (\lceil v(t_j|t_i) \rceil, 0), \tag{11}$$

where $\lceil x \rceil$ is the nearest integer to x upwardly.

Proof 1 *Suppose that player j utilizes $b^*(t_j)$. Any attempt to win 2 units for player i would make her second unit bid set the price. And since the bid from player j is $b_1^*(t_i) \geq \lceil v(t_j|1) \rceil = 4$, player i must bid at least 5 to win. The payoff for using $b^*(t_i)$ is the expected value minus the price paid, which is zero, hence $\pi^* = t_i + 7/2$, while the expected value for using the alternative strategy would be $\pi' \leq 2(t_i + 7/2 - 5)$. Then, we have that $\pi' > \pi^*$ implies (at best) $2(t_i + 7/2 - 5) > (t_i + 7/2) \Rightarrow t_i > 6$, which is impossible.*

As a matter of fact, when there are two bidders, any bid above $\lceil v(t_j|t_i) \rceil$ on the first unit is an equilibrium bid. In an IPV setting, Levin (2005) has shown that any

[8] Lebrun and Tremblay (2003) give a more general proof of this result when values are private.

bid weakly above the upper endpoints of the distribution, if the reservation price is zero, is an equilibrium. This is indeed true also in this model, but the proposed equilibrium risk dominates all other equilibria.

But there also exist other equilibria. If both bidders bid 1, 2 or 3 on the second unit, irrespective of t_i, the bids also become equilibrium bids. But since it is highly unclear on which of these equilibria the subjects would coordinate, the zero-bid on the second unit is focal as well as payoff-dominating in undominated strategies.

When there are more than two players in the game, two things happen. First, we have to correct downwards instead of upwards, as above, because now there is a chance that someone's first-unit bid may become the price-setting bid. And, second, as a result of the first, the zero bid on the second unit is no longer an equilibrium. This is the case since there are now at least three bidders and two units, and all three bidders have a weak incentive to bid the true (expected) value of the first unit.

Conjecture 7 (More than two players) *When there are more than two bidders in the auction, it is an undominated strategy to bid the following on the first unit:*

$$b_1^*(t_i) = \lfloor v(t_j|t_i) \rfloor. \tag{12}$$

where $\lfloor x \rfloor$ is defined as the nearest integer of x downwardly.

Proof 2 *First, note that to bid more than b_1^* will incur an expected loss if the bid is above both $\lfloor v(t_j|t_i) \rfloor$ and the price. That is, suppose that player i bids $b_1' > \lfloor v(t_j|t_i) \rfloor$. Then if $b_1' > p > \lfloor v(t_j|t_i) \rfloor$, a loss of $p - \lfloor v(t_j|t_i) \rfloor$ will be realized on that unit.*

Second, suppose that the bid is below the equilibrium bid $b_1' < b_1^*$. Then, three cases appear; first, if the bid is below the value which, in turn, is weakly below the price, i.e. $p \geq \lfloor v(t_j|t_i) \rfloor > b_1'$, then nothing would change if the player were to raise the bid to $\lfloor v(t_j|t_i) \rfloor$. Next, if the bid is below $\lfloor v(t_j|t_i) \rfloor$ and above the price, $\lfloor v(t_j|t_i) \rfloor > b_1' > p$, nothing would change here either if the bid was increased to $\lfloor v(t_j|t_i) \rfloor$. The last case is if the value is greater than the price and the price is weakly greater than the bid, $\lfloor v(t_j|t_i) \rfloor > p \geq b_1'$. Now, if the player raised the bid to $\lfloor v(t_j|t_i) \rfloor$, she would win a unit at a more profitable price. Thus, to bid the proposed equilibrium bid on the first unit is (weakly) dominant in expectation.

Now, by the last conjecture, when there are more than two players, the bid on the second unit will be weakly bounded from below by the first-unit bid from the low type player. Thus

Conjecture 8 (More than two players) *The second unit bid is weakly bounded by 4, i.e. $b_2(t_i)^* \geq b_1^*(1) = 4$.*

Proof 3 *If player i, say, bids below 4, she will win at most one unit and get the payoff: $\pi_i' = (t_i + 7/2 - p)k_i'$, where $k' \leq 1$. If the player bids 4, the payoff will be: $\pi_i^* = (t_i + 7/2 - p)k_i^*$, where $k_i^* \geq k_i'$ since the bid $b_2(t_i)^*$ now competes against the* *other bids, which the zero bid did not. And, since the bid does not affect the price, p will be the same in both payoff functions above. Hence, $\pi_i^* \geq \pi_i'$.*

Conjecture 9 (Many players) *The more bidders in the auction, the higher the bids. This is true for both the first-unit bid and the second-unit bid.*

Proof 4 *Given any realization of the two dice, we see from equation (6) that the conditional expected value weakly increases with the number of players. Besides, as can be seen from conjectures 7 and 8, since both the first-unit bid and the second-unit bid are dependent on that value, we have that both bids increase with the number of players.*

9.5.3 The Vickrey auction

In the Vickrey auction, a player who wins k_i units pays the k_i highest losing bids of the other players - that is, the k_i highest losing bids not including her own. Hence, the winner is asked to pay an amount equal to the externality she exerts on other competing bidders.

The Vickrey auction is known to have an *ex post equilibrium*, or a *no-regret equilibrium*. That is, an ex post equilibrium is a Bayesian equilibrium with the additional requirement that even if all players' signals were known to a particular bidder, it would still be optimal for her not to alter her strategy, that is, she would not suffer from any regret. [9] This Bayesian strategy is:

$$b^*(t_i) = (t_i + \lceil \frac{t_i}{2} \rceil + 2, t_i + \lceil \frac{t_i}{2} \rceil + 1),$$

where $\lceil \cdot \rceil$ is defined as the nearest integer upwardly. This is indeed an equilibrium:

Proof 5 *If the type-t_i bidder bids less, the number of units that she wins is at most what she would win by bidding $b^*(t_i)$. For any of the units won, the prices will be the same as before, but she will forgo some surplus for units that she did not win.*

If she instead bids $b(t_i) > b^*(t_i)$, then she wins at least as many units as before. The prices for the first k_{t_i} units will remain the same as if she bid $b^*(t_i)$. For any additional units, however, the price paid will be too high, since for $k > k_{t_i}$ the price is greater than the value for the item(s).

But, as in the uniform auction, we have an extreme demand reduction strategy. This equilibrium is the same as in the uniform auction, i.e.:

$$b^*(t_i) = (\lceil v(t_i|t_i) \rceil, 0). \tag{13}$$

[9] For this to be true in this setting, we must have that the value function satisfies what Ausubel (1999) calls *value monotonicity* and *value regularity*. This, indeed, is true for the value function in this paper.

Proof 6 *The proof is as in the uniform auction, hence it is omitted.*

But this demand reduction strategy is a much weaker equilibrium strategy in the Vickrey auction than in the uniform auction, because, if player i bids the above strategy in the uniform auction, player j's best response is to bid the same. That is not entirely true in the Vickrey auction since you never pay what you bid, but what the other bids. Hence, in the Vickrey auction, player j can bid any number below her conditional expected value for the second unit and still be an equilibrium strategy. And, by the same token, any bid below the conditional expected value is an equilibrium bid. For this equilibrium, as in the uniform auction, we have that any bid on the first unit above the conditional expected value is an equilibrium bid.

9.6 Expected revenue

In a (pure) common value auction, revenue is strongly negatively correlated with profit. And seen above, both the uniform and the Vickrey auctions have equilibria that give the entire surplus to the buyer, which is the same as the expected value of the two integers, i.e. 7. This translates into zero revenue to the seller.

The discriminatory auction, on the other hand, has a unique equilibrium, and to find the expected revenue, we calculate the probability for each set of possible joint signals between the players. Then, we make use of the strategies implicitly inherent in the signals to compute the price paid for each possible set of joint signals. Then, we have the expected revenue as the product of the intersection of the signals times the realized price in that outcome. For two players, player i and player j, it becomes:

$$E[R] = P(t_i \cap t_j)p(b_i, b_j),$$

where $p(b_i, b_j)$ is the price paid. If there are three players, we instead calculate $P(t_i \cap t_j \cap t_k)p(b_i, b_j, b_k)$, and so on.

By doing this computation, we have for the discriminatory auction:

$$\begin{aligned} E[R^D(2\text{ Players})] &= 11.22 \\ E[R^D(3\text{ Players})] &= 12.38 \\ E[R^D(4\text{ Players})] &= 12.63. \end{aligned} \tag{14}$$

As we already have stated, both the uniform and the Vickrey auction give zero revenue:

$$E[R^{U,V}(2\text{ Players})] = 0 \tag{15}$$

10 Appendix B

Variable	**OLS**	**Robust standard error**
2-player groups	Reference	
3-player groups	1.85***	0.31
4-player groups	3.80***	0.33
Vickrey auction	1.53***	0.33
Uniform auction	1.80	0.32
Design 1	0.02	0.26
Intercept	11.40***	0.28
No of observations	766	
R^2	0.181	

Notes: a; Dependent variable is revenue (price).

b; ***, ** and * denote difference from zero at the one, five and ten percent significance level respectively.

Table 9
Regression on revenue (price)

11 Appendix C

Bidder instructions for the uniform, common value auction:

11.1 Introduction

Hello and welcome. You will participate in an experiment on economic decision-making. The purpose is to study sales by bidding, i.e. through an auction.

You have the opportunity to win money through participation. The show-up fee is SEK 100 (€10), and by learning the rules of the game you have the opportunity to earn more than that. On the other hand, you could also lose in the process. To ensure that you walk away with at least SEK 100 in your pocket, we give you a starting balance of SEK 50. If you lose this money, you will be excluded from the experiment. Your winnings, and the show-up fee, will be paid in cash after the experiment.

A rule that applies at all times is that all communication between participants is prohibited. If you have any questions, raise your hand and I will come to you and you may ask your question in a whisper. If I believe the question must be answered, I will repeat it to everyone and give the answer.

11.2 Design

Rounds: The experiment consists of several rounds. In each round, 2 identical objects, or units, are to be sold through an auction. (How many rounds to actually be played will be unknown to you.)

The commodities: We will name the units as unit A and unit B. Each of you has a value associated with owning these units and would like to buy them. We call this the redemption value, which is the same for both units.

The redemption value: Before the start of each round, the value of the units is randomly determined through the roll of two dice. The redemption value will then be the sum of the dice. The value can thus never be less than 2, and the maximum is 12. Therefore, the (value) v belongs to the set $\{2, 3, \cdots, 11, 12\}$. However, you will not know what this value is. Instead, you will get private information about this value.

Information: Your information will consist of one of the dice; the other die will be hidden. Thus, you have to make your bids with only partial information of the value. The program randomizes which of the two dice you will see. Other players may, but must not, see the same die as you do.

Opponents: Before the beginning of each round, the program will randomly choose how many players you will be matched with. You can have one, two or three opponents. Your group-size will be seen on your screen.

Bids: After receiving your information, that is, after seeing your die, you should decide on what you want to bid for the units. You are permitted to place equal or different bids for the units.

11.3 Instructions

Buy: Those who have placed the highest bid, and the next-to-highest bid, purchase the units. This may be the same person or two different people. If there are ties among the (winning) bids, the program will randomly choose the winner(s).

Price: The winners will pay a price equal to the highest bid that does not win, That is, the highest bid that is rejected. All winners pay the same price for the units.

Example: 2 units are sold. Three people (A, B, C) have the three highest bids: 10 (A), 9 (B), 8 (C). A and B purchase the units, and both pay 8.

Gain/Loss: The winners make a profit equal to the difference between the (redemption) value and the price. If the difference is negative, a loss is the result.

Example of profit: You won one unit, and the price was 6. The value of the unit was 8. You made a profit of 2 ($8 - 6 = 2$).

Example of a loss: You won one unit, and the price was 10. The value of the unit was 8. You then made a loss of 2 ($8 - 10 = -2$).

Note If you do not have one of the highest bids, nothing happens. The profit is zero.

11.4 Practical execution

Bidding: You will come to a (web-)page where you see two dice, one of them without dots. The one with dots is your signal. Below the dice, there will be 2 fields, one for unit A and one for unit B. You place your bids for the two units in these fields. Only integers between 0 and 12 are possible. (The units are identical, and each bid is for one of the two units.)

Money: You will see what your current balance is before every game starts on the screen. The starting balance is 10 experimental currency. These will be converted to SEK 5/1 at the end of the experiment. If you lose your starting balance, the auction is over for you.

Lost starting balance: If someone (or some) loses her starting balance, she will no longer participate in the auction. This means that there will be one (or more) person(s) less in the auction. If that happens, the auction continues as usual without them but, since we need to have even groups, the program randomizes which players are going to play in subsequent rounds. You may have to pass a round or two. You will be given notice about that on your screen.

One round: After you enter your bids in the fields, press the button "Add bids". When everyone has pressed the button, bids are ranked. Those who have placed the highest bids purchase units at a price that is determined by the pricing rule for each auction.

If there are more winning bids than units for sale, the program randomizes the winners. The balance is recalculated and a new round starts. On the screen you will see the redemption value for the units, the price, the winning bids, own won units, and own profits/losses.

The end: After a certain number of rounds, the experiment will end and you will come to a page showing what you have earned in the experiment.

11.5 Summary

- You will play a certain number of rounds and, in each round, two identical units are for sale.
- You will play against one, two or three opponents. On the screen you will see the number of opponents you have in the current round.
- In each round, all players in an auction have the same redemption value for both units.
- Each player only gets an informational signal about the true value. Subjects may or may not see the same information as their opponents.
- You place two bids, one for each unit. You are allowed to place equal or different bids on the units.
- You start with 10 experimental currency. If you lose this, the experiment is finished for you, and you are excluded from the experiment. But you can also earn more, depending how you and your opponents act.

Bidder Instructions for the discriminatory and Vickrey auctions:

The item price in the Instructions above is changed for the two other auction formats; for the discriminatory auction it is:

Price: The winners pay a price equal to their own placed bid.

Example: 2 units are sold. Three people (A, B, C) have the three highest bids: 10 (A), 9 (B), 8 (C). A and B purchase the units, and they pay 10 and 9, respectively.

And for the Vickrey auction, we have:

Price: The winner(s) pays a price equal to the highest bid that does not win, not including his own. That is, the highest bid that is rejected and comes from someone else.

Example 1: 2 units are sold. Four people (A, B, C, D) have the four highest bids: 10 (A), 9 (B), 8 (C)and 7 (D). A and B purchase the units, and both pay 8.

Example 2: 2 units are sold. Three people (A, B, C) have the four highest bids: 11 (A), 10 (B_1), 9 (B_2), 8 (C). A and B purchase one unit each, A pays 9, and B pays 8 (since 9 is his bid).

Example 3: 2 units are sold. Three people (A, B, C) have the four highest bids: 7 (A_1), 6 (A_2), 5 (B), 4 (C). A purchases both units; for the first he pays 5 and for the second he pays 4.

Otherwise, the instructions for the formats are the same.